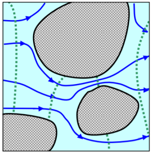
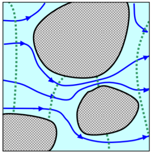
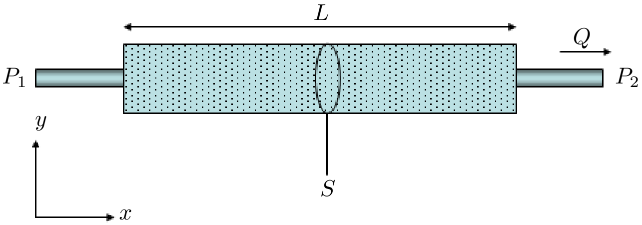


Hydrodynamic transport in a porous media



Olivier Liot Petit

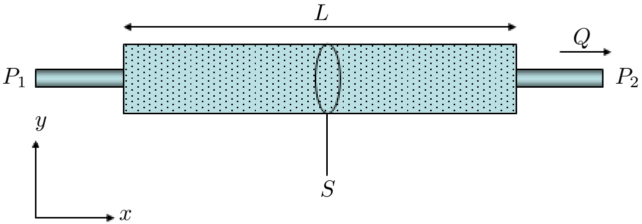
Université de Toulouse
TOULOUSE INP N7
IMFT INSTITUT DE MECANIQUE DES FLUIDES DE TOULOUSE



L
Q
P_1
P_2
y
S
x

Take your smartphone

Flow rate in a porous media



#QDLE#Q#AB*C##

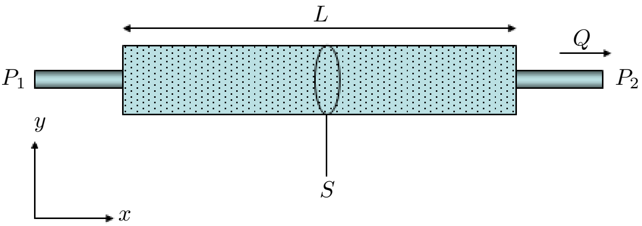
In your opinion, the flow rate is proportional to...

A. $Q \propto \frac{P_1 - P_2}{\eta S L}$

B. $Q \propto \frac{S(P_1 - P_2)}{\eta L}$

C. $Q \propto \frac{L(P_1 - P_2)}{\eta S}$

Flow rate in a porous media



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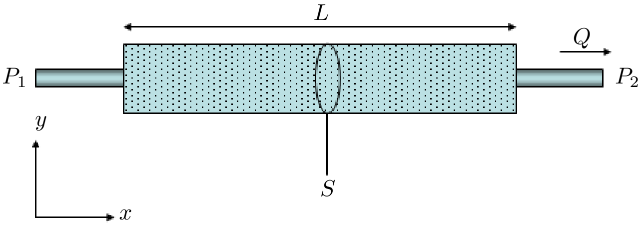

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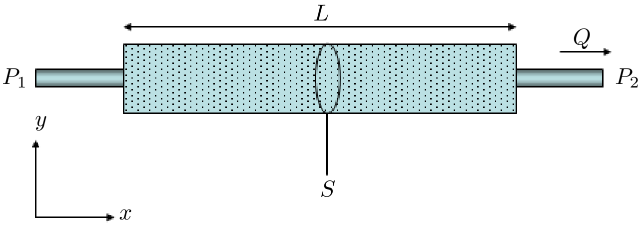
Exercise: fluid velocity in a cylinder assembly



#QDLE#Q#ABC*##

In your opinion, the flow rate is proportional to...

A. $Q \propto \dfrac{P_2 - P_1}{\eta S L}$

B. $\boldsymbol{Q \propto \dfrac{S(P_2 - P_1)}{\eta L}}$

C. $Q \propto \dfrac{L(P_2 - P_1)}{\eta S}$

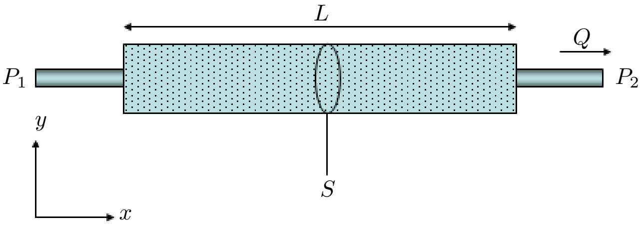
Exercise: fluid velocity in a cylinder assembly

The mean velocity through an assembly of N cylinders (diameter D, length L) is...

A. $U = \dfrac{\epsilon^2}{2a^2} \dfrac{\Delta P}{\eta L}$

B. $U = \dfrac{\epsilon^3}{2a^2(1-\epsilon)} \dfrac{\Delta P}{\eta L}$

C. $U = \dfrac{\epsilon^3}{2a^2} \dfrac{\Delta P}{\eta L}$



In your opinion, the flow rate is proportional to...

A. $Q \propto \dfrac{P_2 - P_1}{\eta S L}$ B. $\boldsymbol{Q \propto \dfrac{S(P_2 - P_1)}{\eta L}}$ C. $Q \propto \dfrac{L(P_2 - P_1)}{\eta S}$

Exercise: fluid velocity in a cylinder assembly

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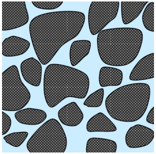
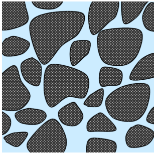
Motivations

Observations:

- > Need the passage from local Stokes' law to a macroscopic law
- > Numerous geometric effects can affect the hydrodynamic transport
- > Possibility of inertial flow



What are the macroscopic laws for hydrodynamic transport in a porous media?



Hydrodynamic transport in porous media:

A. **Derivation of the Darcy's law and permeability computation**

B. **Corrections to Darcy's law**

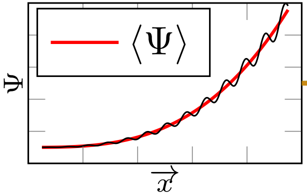
Hydrodynamic transport in porous media

At the end of these lectures, you should be able to:

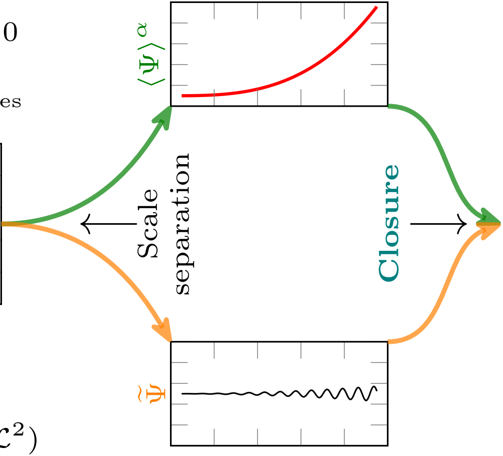
- summarize and interpret the Darcy's law
- compute the Darcy's law using a step-by-step guide
- compute an estimation of the permeability of a porous media
- cite some experimental methods to measure permeability
- define the Klinkenberg effect
- apply the Darcy's law without neglecting inertia (Ergun's law)
- choose the good approach to assess the hydrodynamic transport in a porous media

$$F(\Psi, p(\overrightarrow{x}), \bar{\bar{k}}(\overrightarrow{x})) = 0$$

$$\begin{cases} p(\overrightarrow{x}): & \text{forcing} \\ \bar{\bar{k}}(\overrightarrow{x}): & \text{media properties} \end{cases}$$



$$F_{\langle \cdot \rangle}(\langle \Psi \rangle^{\alpha}, \widetilde{\Psi}, \overrightarrow{x}, \bar{\bar{k}}(\overrightarrow{x}))) = 0$$



$$F_{\tilde{\cdot}}(\langle \Psi \rangle^{\alpha}, \widetilde{\Psi}, \overrightarrow{x}, \bar{\bar{k}}(\overrightarrow{x}))) = 0$$

$$F_H(\langle \Psi \rangle^{\alpha}, \langle p(\overrightarrow{x}) \rangle, \bar{\bar{k}}_{eff}) = 0$$

Closure

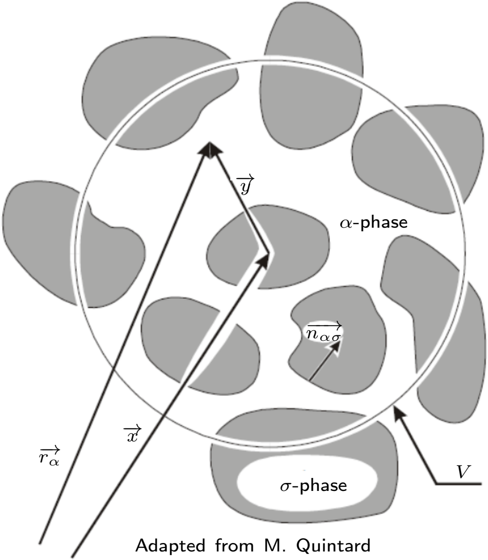
$$\widetilde{\Psi} = g(\langle \Psi \rangle^{\alpha}) + \mathcal{O}(r^2/\mathcal{L}^2)$$

$$\begin{cases} g: & \text{linear relation} \\ \Rightarrow \bar{\bar{k}}_{eff}: & \text{effective properties} \end{cases}$$

Volume-averaging method

- Establish local equations
- Give local and macroscopic boundary conditions
- Volume averaging on continuity equation
- Volume averaging on momentum equation (pressure then viscous term)
- Closure problem
- Approximations to obtain the Darcy's law

Exercise: first steps of Darcy's law derivation



Adapted from M. Quintard

> Gravity is neglected, laminar and stationnary flow

Local equations

$$\eta \Delta \overrightarrow{v} = \overrightarrow{\mathrm{grad}} P \quad (0.1)$$

$$\mathrm{div}\, \overrightarrow{v} = 0 \quad (0.2)$$

Boundary conditions

$$\overrightarrow{v} = \overrightarrow{0} \quad \text{at} \quad S_{\alpha\sigma} \quad (0.3)$$

$$\overrightarrow{v} = \overrightarrow{f}(\overrightarrow{r}, t) \quad \text{at} \quad S_{\alpha e} \quad (0.4)$$

Reminder: volume averaging theorems

Theorem 1 – Gradient spatial averaging

$$\langle \overrightarrow{\mathrm{grad}} \Psi \rangle = \overrightarrow{\mathrm{grad}} \langle \Psi \rangle + \frac{1}{V} \int_{S_{\alpha\sigma}} \Psi \overrightarrow{n_{\alpha\sigma}} dS$$

Theorem 2 – Divergence spatial averaging

$$\langle \mathrm{div} \overrightarrow{\Psi} \rangle = \mathrm{div} \langle \overrightarrow{\Psi} \rangle + \frac{1}{V} \int_{S_{\alpha\sigma}} \overrightarrow{\Psi} \cdot \overrightarrow{n_{\alpha\sigma}} dS$$

- Explication of the term $\frac{1}{V} \int_{S_{\alpha\sigma}} \Psi \overrightarrow{n_{\alpha\sigma}} dS$ (interfacial integral)
- $\Psi = \langle \Psi \rangle^{\alpha} + \tilde{\Psi}$ (average + fluctuations)

Under scale separation assumption, we have:

$$\frac{1}{V} \int_{S_{\alpha\sigma}} \Psi \overrightarrow{n_{\alpha\sigma}} dS = -\langle \Psi \rangle^{\alpha} \overrightarrow{\mathrm{grad}} \epsilon_{\alpha} + \frac{1}{V} \int_{S_{\alpha\sigma}} \tilde{\Psi} \overrightarrow{n_{\alpha\sigma}} dS$$

Take your smartphone

In what does the continuity equation result on the intrinsic average velocity?

A. $\mathrm{div}\langle\overrightarrow{v}\rangle^{\alpha} = 0$

B. $\mathrm{div}\langle\overrightarrow{v}\rangle^{\alpha} = \epsilon_{\alpha}^{-1}\overrightarrow{\mathrm{grad}}\epsilon_{\alpha}\cdot\langle\overrightarrow{v}\rangle$

#QDLE#Q#ABC*##

C. $\mathrm{div}\langle\overrightarrow{v}\rangle^{\alpha} = -\epsilon_{\alpha}^{-1}\overrightarrow{\mathrm{grad}}\epsilon_{\alpha}\cdot\langle\overrightarrow{v}\rangle$

In what does the continuity equation result on the intrinsic average velocity?

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C. $\mathrm{div}\langle\overrightarrow{v}\rangle^{\boldsymbol{\alpha}} = -\boldsymbol{\epsilon}_{\boldsymbol{\alpha}}^{-1}\overrightarrow{\mathrm{grad}}\boldsymbol{\epsilon}_{\boldsymbol{\alpha}} \cdot \langle\overrightarrow{v}\rangle$

> $\langle\overrightarrow{v}\rangle = \epsilon_{\alpha}\langle\overrightarrow{v}\rangle^{\alpha}$

Take your smartphone

After averaging, what is the relation for the pressure?

A. $\langle \overrightarrow{\text{grad}} P \rangle = \frac{1}{V} \int_{S_{\alpha\sigma}} \widetilde{P} \overrightarrow{n_{\alpha\sigma}} dS$

B. $\langle \overrightarrow{\text{grad}} P \rangle = \epsilon_\alpha \overrightarrow{\text{grad}} \langle P \rangle^\alpha + \frac{1}{V} \int_{S_{\alpha\sigma}} \widetilde{P} \overrightarrow{n_{\alpha\sigma}} dS$

#QDLE#Q#AB*C##

C. $\langle \overrightarrow{\text{grad}} P \rangle = \epsilon_\alpha \overrightarrow{\text{grad}} \langle P \rangle^\alpha - \frac{1}{V} \int_{S_{\alpha\sigma}} \widetilde{P} \overrightarrow{n_{\alpha\sigma}} dS$

Momentum equation

After averaging, what is the relation for the pressure?

A. $\langle \overrightarrow{\mathrm{grad}} P \rangle = \frac{1}{V} \int_{S_{\alpha\sigma}} \widetilde{P} \overrightarrow{n_{\alpha\sigma}} dS$

B. $\boldsymbol{\langle \overrightarrow{\mathrm{grad}} P \rangle = \epsilon_\alpha \overrightarrow{\mathrm{grad}} \langle P \rangle^\alpha + \frac{1}{V} \int_{S_{\alpha\sigma}} \widetilde{P} \overrightarrow{n_{\alpha\sigma}} dS}$

C. $\langle \overrightarrow{\mathrm{grad}} P \rangle = \epsilon_\alpha \overrightarrow{\mathrm{grad}} \langle P \rangle^\alpha - \frac{1}{V} \int_{S_{\alpha\sigma}} \widetilde{P} \overrightarrow{n_{\alpha\sigma}} dS$

- $\langle P \rangle = \epsilon_\alpha \langle P \rangle^\alpha$
- $P = \langle P \rangle^\alpha + \widetilde{P}$

Darcy's law

- Relation giving a proportionality between surface average velocity and pressure gradient (**closure**)
- Homogeneous porous media

Darcy's law expression

$$\langle \vec{v} \rangle = -\frac{\overline{\overline{K}}}{\eta} \overrightarrow{\mathrm{grad}} \langle P \rangle^{\alpha}$$

Permeability tensor $\overline{\overline{K}}$ properties

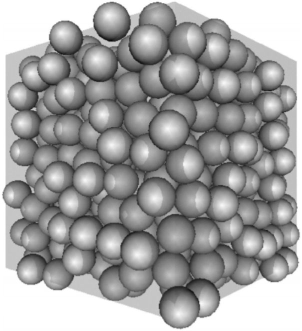
- 3×3 matrix, positive elements
- Symmetric matrix from Onsager reciprocity
- Scalar for an isotropic porous media
- Dimension of an area (m^2)

Some typical permeabilities

Media	Permeability (cm^2)	Media	Permeability (cm^2)
Gravel	$10^{-3} - 10^{-4}$	Oil reservoirs rocks	$10^{-7} - 10^{-9}$
Fine sand	$10^{-8} - 10^{-11}$	Granite	$10^{-14} - 10^{-15}$

- Specific unit: darcy ($1\,\mathrm{d} = 9.87 \times 10^{-13}\,\mathrm{m}^2$)

Exercise: computation of some permeabilities



Exercise: computation of some permeabilities

What is the permeability of a packing bed of spherical particles?

A. $k \approx \dfrac{\epsilon^3 D^2}{180(1-\epsilon)^2}$

B. $k \approx \dfrac{\epsilon^3 D^2}{18(1-\epsilon)^2}$

C. $k \approx \dfrac{\epsilon^3 D^2}{180(1-\epsilon)}$

#QDLE#Q#A*BC##

Permeabilities of some classical porous media

Exercise: computation of some permeabilities

What is the permeability of a packing bed of spherical particles?

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- Kozeny-Carman relation (very important for sand-style porous media)
- For a Hele Shaw cell: $k = b^2/12$

Remember:

- For a capillaries-like porous media: $k = \frac{\epsilon^3 D^2}{2\tau^2 a^2}$

Experimentally: sand bed, $D = 80\,\mu$m, $\epsilon = 0.4 \Rightarrow k = 6500\,$mD

Permeabilities of some classical porous media

Exercise: computation of some permeabilities

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#QDLE#Q#ABC*#30#

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Experimentally: sand bed, $D = 80\,\mu$m, $\epsilon = 0.4 \Rightarrow k = 6500\,$mD

What is the theoretical permeability value ($1\,$d$= 9.87 \times 10^{-13}\,$m^2)?

A. 3800 md B. 0.64 d C. 6400 md

Exercise: computation of some permeabilities

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What is the theoretical permeability value ($1\,$d$= 9.87 \times 10^{-13}\,$m^2)?

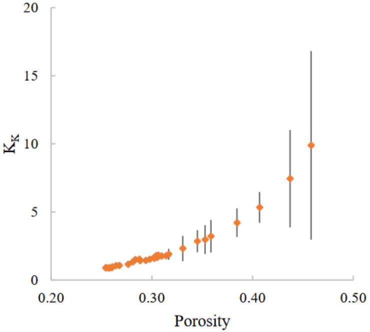
A. 3800 md B. 0.64 d C. **6400 md**

- Good agreement with experiment

- More general Kozeny-Carman expression :

$$k = \frac{\epsilon^3 D^2}{36 K_k (1-\epsilon)^2}$$

- K_k is called the Kozeny's coefficient.
- Depends on the particles arrangement



From Valencia Navarro (2020)

- Example of Kozeny coefficient depending on the porosity for polydisperse particles

Exercise: Hydrodynamic of yeast clog

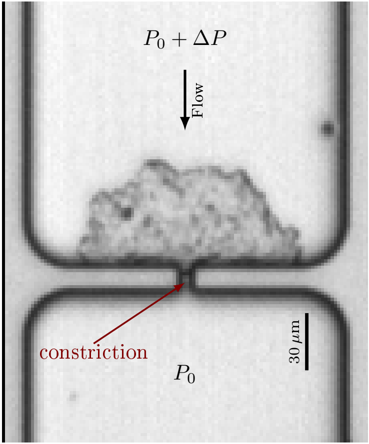


Figure: Yeasts assembly trapped by a microfluidic constriction. A pressure drop ΔP is applied and the suspension flows from top to bottom.

- Relation giving a proportionality between surface average velocity and pressure gradient
- Homogeneous porous media

Darcy's law expression

$$\langle \vec{v} \rangle = -\frac{\overline{\overline{K}}}{\eta} \overrightarrow{\text{grad}} \langle P \rangle^{\alpha}$$

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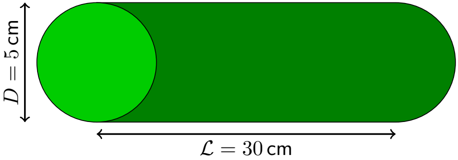
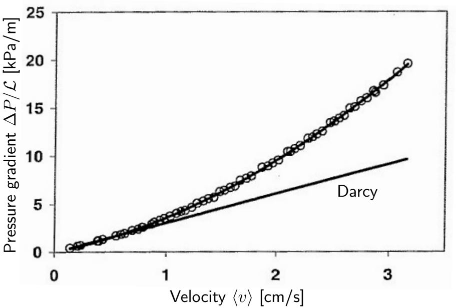
Underlying hypothesis:

- Local physics law still valid at pore scale
- Negligible inertia

> Experimental observation





> Quartz sand, diameter $d = 3\,\text{mm}$

Apparent permeability lower than the expected one

Reynolds number at pore scale

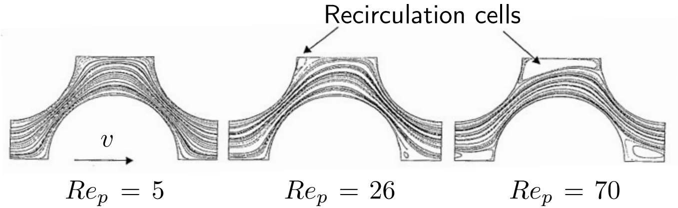
$$Re_p \approx \frac{\rho \langle v \rangle \ell_\alpha}{\eta \epsilon}$$

Numerical application:

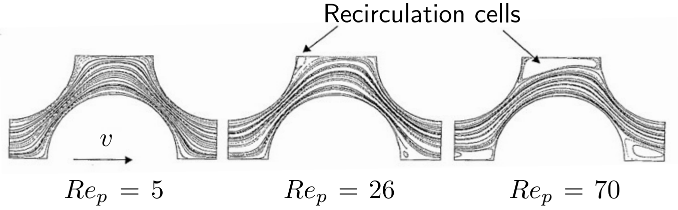
> $\langle v \rangle = 3\,\text{cm/s}$, water, $\ell_\alpha = d$

> $Re_p \approx 225$

Inertia effects: phenomenological explanation



- Recirculation cells $\Rightarrow$ effective section reduction
- Permeability reduction



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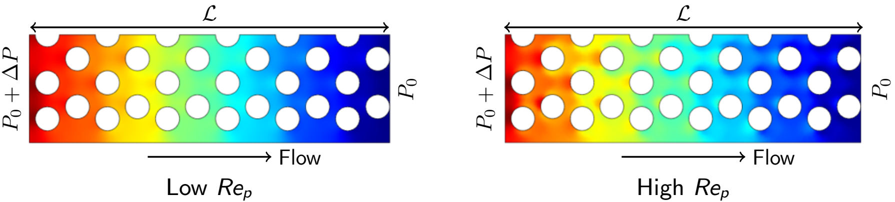
- Permeability reduction

COMSOL simulations



- Changes in the pressure field (especially near obstacles)

Inertia effects: Darcy-Forchheimer's law

- Inertia taken into account for $Re_p > 1 - 10$
- Appearance of inertial term in NS equation: $(\overrightarrow{v} \cdot \overrightarrow{\mathrm{grad}}) \cdot \overrightarrow{v}$

Using homogenization technique(s) we get:

Darcy-Forchheimer's law

$$\overrightarrow{\mathrm{grad}} \langle P \rangle^{\alpha} = -\eta \overline{\overline{K}}^{-1} \langle \overrightarrow{v} \rangle - \rho \beta \langle \overrightarrow{v} \rangle^{2} \text{ with } \beta\text{: inertia coefficient } [\mathrm{m}^{-1}]$$

- β independent of the fluid, depends only on the porous media
- Local law: $\overrightarrow{\mathrm{grad}} P = -\eta \overline{\overline{K}}^{-1} \overrightarrow{v} - \rho \beta || \overrightarrow{v} || \overrightarrow{v}$

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- Local law: $\overrightarrow{\mathrm{grad}} P = -\eta \overline{\overline{K}}^{-1} \overrightarrow{v} - \rho\beta ||\overrightarrow{v}|| \overrightarrow{v}$

Apparent permeability

$$\langle \overrightarrow{v}\rangle = -\frac{\overline{\overline{K}}_{app}}{\eta}\overrightarrow{\mathrm{grad}}\langle P\rangle^{\alpha} \text{ with } K_{app} = \frac{\overline{\overline{K}}}{1 + \epsilon\beta\sqrt{\overline{\overline{K}}} Re_p^*}$$

- $Re_p^* = \dfrac{\rho\sqrt{\overline{\overline{K}}}||\langle \overrightarrow{v}\rangle||}{\eta\epsilon}$

- Case of a particles packed bed
- Including inertia effects

Ergun's law

$$\frac{\Delta P}{L} = \frac{180(1-\epsilon)^2}{\epsilon^3}\frac{\eta}{D_p^2}U + \beta\frac{3\,(1-\epsilon)}{4\,\epsilon^3}\frac{\rho}{D_p}U^2$$

- β: determined experimentally

	β
Ergun	2.33
Macdonald (smooth)	2.40
Macdonald (rough)	5.33

- Valid only for non-consolidated porous media

Study of an article

Gas Flow in Porous Media with Klinkenberg Effects (Wu *et al.*, 1998)

Connect to the Moodle platform (course Transfers in porous media – MIPO)