DÉPARTEMENT MFEE
3ème année Parcours FEP/M2 FEIP
Toulouse INP – ENSEEIHT

TRANSFERS IN POROUS MEDIA
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Examination

Instructions and recommendations

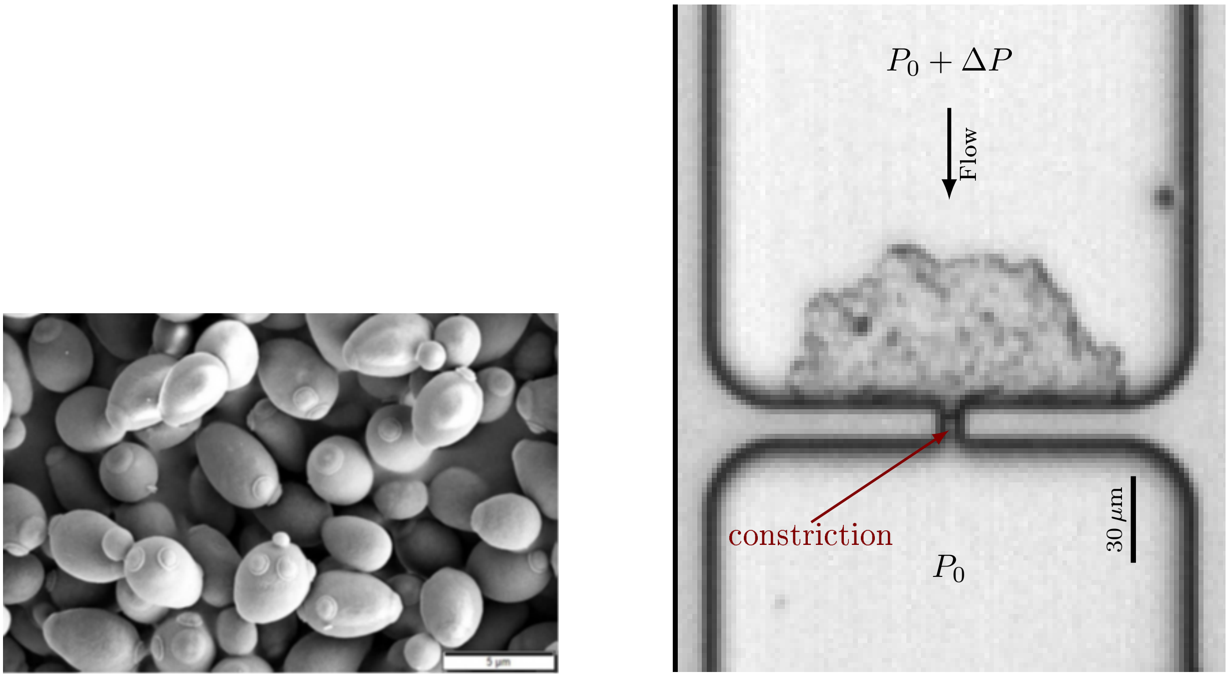
1. You may read these instructions, but do not turn the page or begin the work until instructed.
2. Lecture notes provided at the end of the lecture are allowed with the examination. You can also have your personal notes. Any electronic device is forbidden.
3. Communication between students is not allowed.
4. Answers must be given in English. Nevertheless, language level will not be a notation criterion.
5. Time limit : 120 minutes. Please stop working when asked.
6. Detail all work and assumptions in order to get the maximal score. A clear and detailed redaction is required.
7. This examination contains two sections. The first one is an exercise. The second one is a guided analysis of an article. Score is balanced between these two parts.
8. This exam is *probably* too long for two hours, so do not worry if you do not finish it.
9. Consequently, you should split equitably your time between both parts of the exam.
10. An article is provided in appendices.
11. The present document contains 6 pages.

Exercise 1 : Bio-reactive transport in a yeast assembly 20 points

This exercise aims at studying the way nutrients are transported in a yeast assembly formed under flow.

A Hydrodynamic resistance

Yeasts are unicellular microorganisms widely present in nature and food industry. For example, baker's yeasts *Saccharomyces cerevisiae* are used to make both bread and beer. It is also a model organism used in a lots of biophysics studies. Their typical diameter is approximately 4.5 μm. Figure 1a show a micrography of yeasts *S. cerevisiae*.



(a) Baker's yeasts observed using scanning electron microscopy.

(b) Yeasts assembly trapped by a microfluidic constriction. A pressure drop ΔP is applied and the suspension flows from top to bottom.

FIGURE 1

Some very recent laboratory experiments consist in generating assembly of yeasts by trapping them in a microfluidic device in order to understand how does it react to various physico-chemical stress. Figure 1b shows a micrography of a yeast assembly during its construction. A suspension of yeasts is flowed in a microfluidic device using a pressure difference ΔP between the inlet and the outlet of the device. Its kinematic viscosity is $\nu = 2 \times 10^{-6}\,\mathrm{m^2/s}$ and its dynamic viscosity is $\mu = 2 \times 10^{-3}\,$Pa.s. The microfluidic device is $h = 6\,\mu$m deep and $W = 140\,\mu$m wide. A constriction of $w = 6\,\mu$m traps the yeasts leading to the construction of a **quasi-2D** assembly of yeasts as you can see in figure 2. This assembly is considered as a porous media of porosity $\varepsilon = 0.1$.

1 pt 1. The constriction is 6 μm deep and 6 μm wide. The inlet flow rate before yeast accumulation reaches around 300 nL/min for $\Delta P = 400$ mbar. Compute the order of magnitude of the Reynolds number. Is the flow laminar ?

1 pt 2. After few seconds, a yeast assembly appears at the constriction. It can be considered as a porous media. For the laminar regime, give the relation between fluid velocity and pressure gradient through this assembly using the notations proposed in the lecture. What is the name of this relation ?

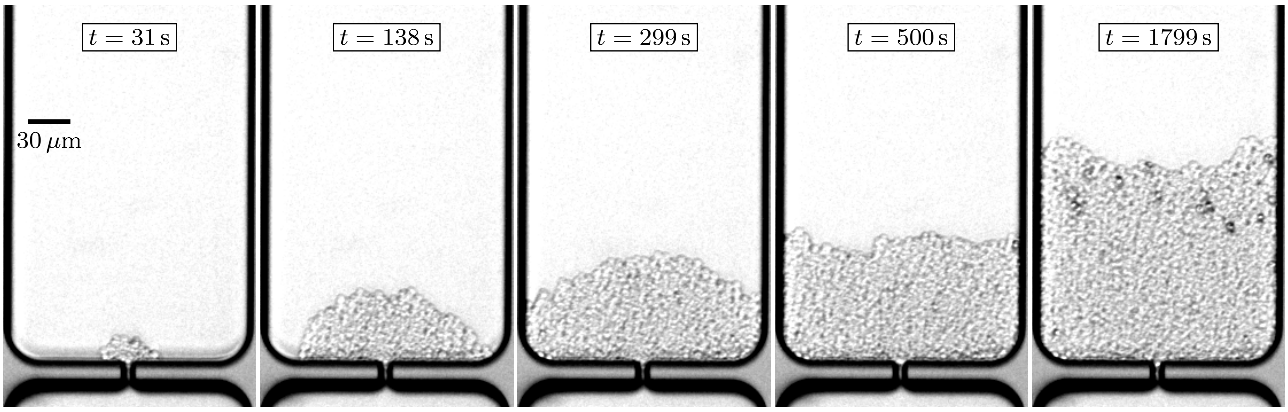


FIGURE 2 – Sequence of pictures showing different steps of yeast assembly construction under flow.

1 pt 3. Remind the hypotheses necessary to get this relation.

1.5 pt 4. Give the definition of both intrinsic average and superficial average. What is the relation between these two averages ?

2 pts 5. Summarize, without formula, the volume-averaging method used to derive the Darcy's law.

1 pt 6. What should be the relation if inertia was not negligible (name and formula) ?

1 pt 7. During the yeast accumulation, the assembly transits from a "cylindrical" regime to a "linear" regime. In the first regime, the yeast assembly can be considered as half a cylinder (height $h = 6\,\mu$m, radius R from the constriction). In the second regime, which is reached when $R \approx W$, it is a parallelepiped (height $h = 6\,\mu$m, width $W = 140\,\mu$m and length L from the constriction). Draw a sketch of each regime including dimensions.

1.5 pt 8. We now consider that the permeability tensor can be reduced to a scalar k. For the first regime, give the relation between $\langle v \rangle$ and the flow rate Q then use it to integrate the Darcy law between $w/2$, the constriction half-width, and R. Show that :

$$Q(R) = \frac{\pi k h}{\mu} \frac{\Delta P}{\ln(2R/w)}. \qquad (1)$$

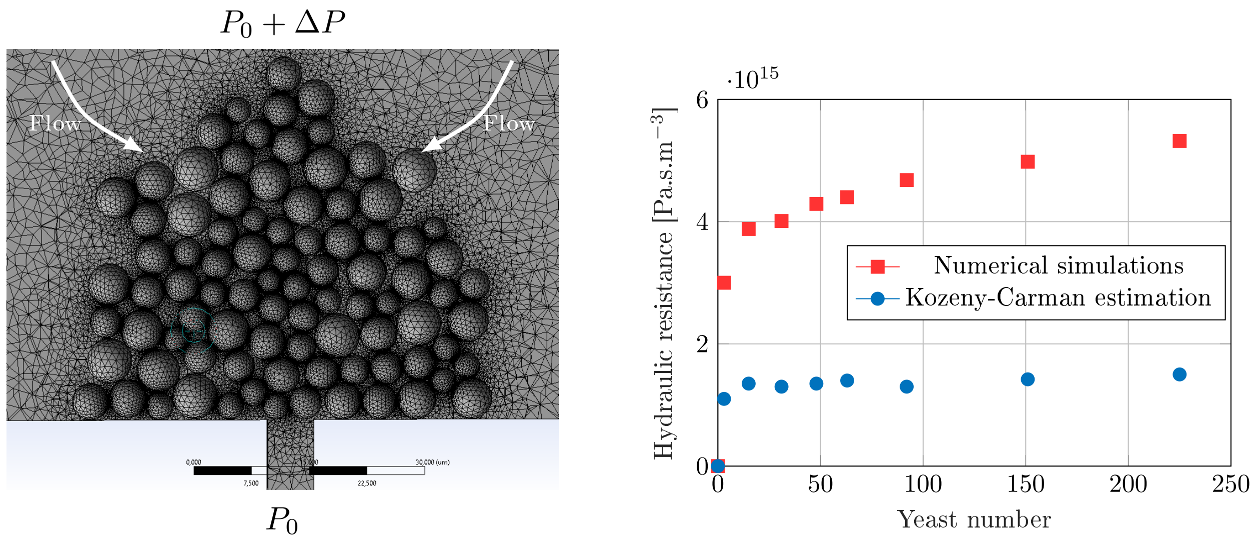
1 pt 9. For the second regime, give the new relation between $\langle v \rangle$ and the flow rate Q then use it to integrate the Darcy law between W, the microfluidic inlet width, and $L \gg W$ the assembly length. You should get a relation between ΔP, Q, L and other parameters of the problem.

1 pt 10. As a first approximation, yeasts can be considered as spherical objects with a diameter of $d_p = 4.5 \pm 0.5\,\mu$m. Give the relation that we demonstrated during the lecture which relates k to ε and d_p. What is its name ?

2.5 pts 11. Figure 3a shows a geometry and mesh used to perform numerical simulations of the flow through an assembly of spherical particles trapped by a constriction (simulations made with *Fluent*). Channel dimensions, particles size distribution and flow conditions are identical to the experiment presented above. Figure 3b shows the hydraulic resistance of the simulated particle assembly for different numbers of particles in the assembly (representing different yeast assembly size). We superposed an estimation of the hydraulic resistance computed using the Kozeny-Carman relation. Comment the plot and discuss the validity of the Darcy's law and of the Kozeny-Carman relation for the configuration studied here.

B Nutrient transport and consumption

Once the assembly is large enough (typically $200\,\mu$m long from the constriction), yeast suspension flow is stopped and replaced by a flow of culture medium, with the same pressure drop ΔP. No more



(a) Geometry and mesh used for the simulations.

(b) Hydraulic resistance of the assembly versus number of particles in the assembly. Numerical simulations and Kozeny-Carman estimations are superposed.

FIGURE 3

yeast arrive in the assembly, but yeasts in the assembly can now grow and divide (we name it cellular proliferation). To do that, yeasts need to be fed by a sufficient quantity of nutrients transported by the culture medium. The consequence is that the assembly keep growing, but due to cell proliferation – no more due to yeast accumulation.

Nutrients are advected by the fluid moving through the assembly then consumed by the yeasts. This problem can be seen as an advection-diffusion problem with a reactive term.

1 pt
1. For this question, we neglect the reactive term due to nutrient consumption. We note $\langle C(z,t)\rangle^\alpha$ the intrinsic average of nutrient concentration in the assembly considered to be long enough to use a 1D representation. Write the volume-averaged advection-diffusion equation with the notations used during the lecture.

0.5 pt
2. Considering that the flow is incompressible, and using the relation $\text{div}(M\overrightarrow{A}) = \overrightarrow{A} \cdot \overrightarrow{\text{grad}}M + M\text{div}\overrightarrow{A}$, show that :

$$\varepsilon \frac{\partial \langle C\rangle}{\partial t} = \varepsilon \text{div}(\overline{\overline{D}} \cdot \overrightarrow{\text{grad}}\langle C\rangle^\alpha) - \langle \overrightarrow{v}\rangle \cdot \overrightarrow{\text{grad}}\langle C\rangle^\alpha. \quad (2)$$

1.5 pt
3. We consider the nutrient consumption during a time scale very short compared to the typical division time scale of the yeasts. This means that we work now in a quasi-static configuration which allows to neglect the temporal variations induced by yeast proliferation. We also consider that the diffusion coefficient can be reduced to a scalar and that the Péclet number is high compared to 1. We remind that the velocity is along $-\overrightarrow{e_z}$ and we note V its norm. We note Y_m the yeast volume concentration in the assembly, considered as homogeneous. The amount of nutrient consumed by time unit by the yeasts can be written as

$$\gamma \left[\mu_{max} \frac{\langle C\rangle^\alpha}{\langle C\rangle^\alpha + K}\right] Y_m. \quad (3)$$

where K, μ_{max} and γ are constant quantities. Under these different assumptions, deduce the resulting bio-reactive transport equation which drives the superficial average of nutrient concentration in the yeast assembly in a quasi-static configuration.

1.5 pt
4. We note C_{inj} the nutrient concentration injected in the assembly at $z = L$ (surface of the assembly). Adimensionalize the previous equation and show that the nutrient concentration profile depends on the dimensionless Damköhler number

$$Da = \frac{\gamma \mu_{max} Y_m L}{C_{inj} V}. \tag{4}$$

1 pt 5. Discuss the possible regimes of concentration profile based on the value of Da.

Exercise 2 : Article analysis 20 points

We propose to study an article entitled "Scalar Mixtures in Porous Media" from Kree & Villermaux. Before beginning the questions, have a quick overview of the article to see its structure and the main topic addressed. **Do not read the whole paper before beginning**, it will be a waste of time. Your answers should be written with your own words, do not paraphrase.

Questions

A Generalities about the article

0.25 pt 1. Which is the year of publication of this article ?

0.25 pt 2. In which journal has it been published ?

2 pts 3. In few lines and without paraphrasing the text, summarize the objectives of the authors.

B Introduction

1.5 pt 1. Which are the different ways for a solute or particles to disperse in a porous media ?

0.5 pt 2. In which journal (give the whole name) and in which year has been published the article which inspired the authors for the visualization *in situ* of the concentration fields in a porous media ?

1 pt 3. Which new studies do the authors propose concerning dispersion properties in heterogeneous media ?

C Setup and Methods

1 pt 1. Describe briefly the interest to adapt refractive index for visualization through a porous media.

0.5 pt 2. What are the media which must have the same index ? What must be the precision on index matching ?

0.5 pt 3. Which property of the beads is used to get this index matching ?

0.5 pt 4. Remind the expression of porosity ϕ.

0.25 pt 5. Which is the other name used for "interstitial velocity" ?

1 pt 6. What does represent the Péclet number ? Will the molecular diffusion have a significant impact in the experiment proposed in the article ?

0.25 pt 7. How many injection sources are present in the setup ?

D Mixing Properties of the Flow

D.1 Plume dispersion

0.5 pt 1. Why is it necessary to define a "perpendicular" diffusion coefficient in this experiment ?

0.5 pt 2. Re-write the expression of the transverse width of the plume as a function of x. What does represent s_0 ?

D.2 Growing sheets and necessary overlap

1 pt 1. Describe using a diagram the analysis method of "lines growth" detailed by the authors.

1 pt 2. Having in mind that $\left\langle \frac{\Delta r}{\Delta t} \right\rangle$ can be written as :

$$\frac{d\left\langle \frac{\Delta r}{\Delta t} \right\rangle}{dr} + \frac{1}{d}\left\langle \frac{\Delta r}{\Delta t} \right\rangle = \gamma, \tag{5}$$

find the expression of equation (4) in the article.

1 pt 3. Give a physical interpretation of this expression.

3 pts 4. Subsections 2 and 3 describe the behaviour of a dye blob in the porous media. Summarize the behaviour of a piece of blob during its displacement in the porous media.

0.5 pt 5. More generally, to what kind of flow the dispersion in porous media can be compared ?

D.3 Entangled fields

0.5 pt 1. What is the system change in this section ?

1.5 pt 2. Describe what represents figure 4 without paraphrasing the text.

1 pt 3. In figure 5, the dashed line $\Gamma_{n=8}(C/2)$ do not superpose to the experimental curve $P_1(C_1)$. What does it highlight ?