
Examination

Instructions and recommendations

1. **You may read these instructions, but do not turn the page or begin the work until instructed.**
2. Lecture notes provided at the end of the class are allowed during the examination. You can also have your personal notes. Any electronic device is forbidden, including calculator.
3. Communication between students is not allowed.
4. **Please**, add a 10-square margin at the top of your first page, and a 4-square margin at the left or right of each page.
5. Answers can be given in English, French or Spanish. Language level will not be a notation criterion.
6. Time limit : 120 minutes. Please stop working when asked.
7. Detail all work and assumptions in order to get the maximal score. A clear and detailed redaction is required.
8. This examination contains two sections. The first one is an exercise. The second one is a guided analysis of an article. Score is balanced between these two parts.
9. This exam is *probably* too long for two hours, so do not worry if you do not finish it.
10. Consequently, you should split equitably your time between both parts of the exam.
11. An article is provided in appendices.
12. The present document contains 5 pages.

Exercise 1 : Let's brew some coffee...

20 points

This exercise aims at modelling different ways to brew coffee. In every section, we consider that coffee is a powder made of spherical particles of diameter $d_p = 0.5$ mm and density ρ_c . The coffee flavours are extracted from the particles in contact with hot water (density $\rho_w = 1000$ kg/m³, viscosity $\eta = 10^{-3}$ Pa.s) to produce the beverage we drink, following different method that we will describe.

A Piston coffee maker

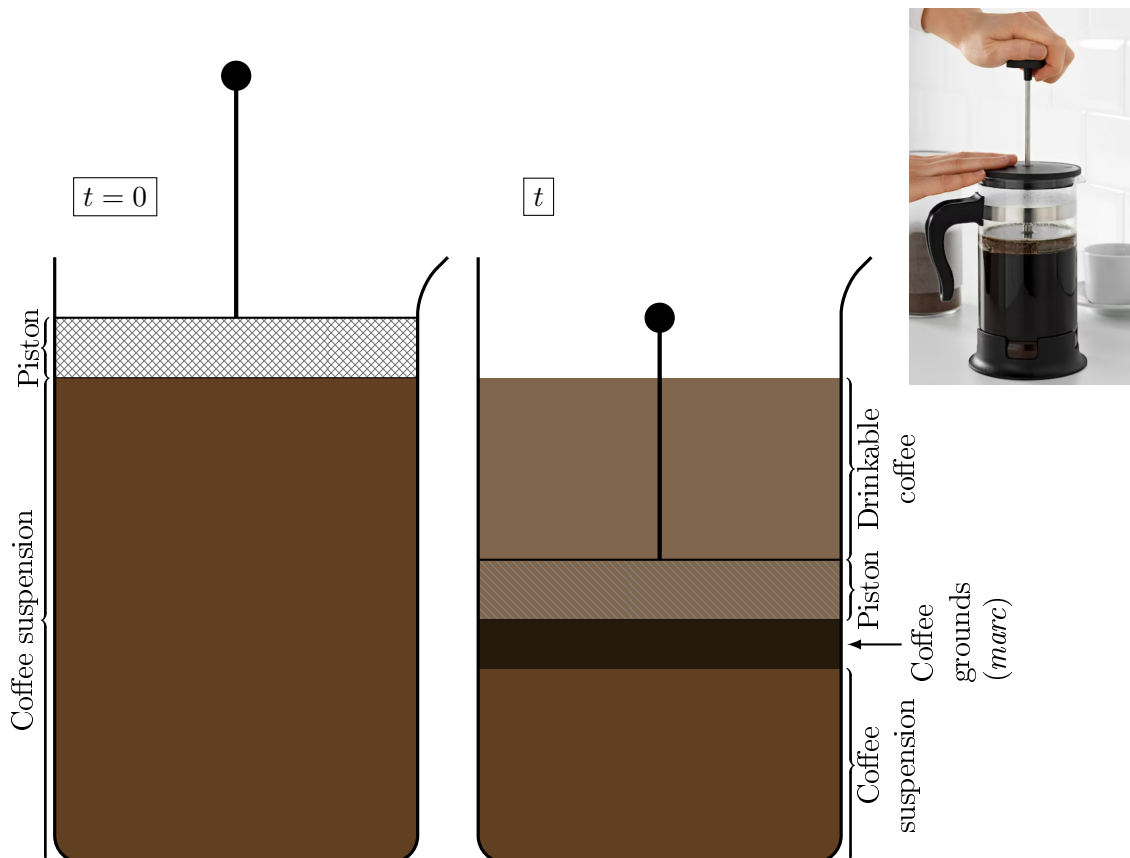


FIGURE 1 – Sketch of a piston coffee maker.

The first example concerns a piston coffee maker. Coffee powder is placed in a tank (cross-section S) then hot water is added. We consider as a first approximation that coffee particles' density ρ_c is the same as hot water's density ρ_w . This leads to a coffee suspension (coffee particles are dispersed in the fluid) with a volume fraction in coffee $\phi = 0.1$.

The tank is closed with a porous piston whose permeability is constant and denoted k_p and thickness is denoted L . The position of the bottom part of the piston is noted z where $z = 0$ corresponds to $t = 0$ (the bottom part of the piston is in contact with the coffee suspension but there is no water inside the piston). z is counted positively downwards.

A constant force F is applied by a human on the handle (the black circle on figure 1). At $t = 0$, the piston is in contact with the coffee suspension and empty of liquid. At time t , coffee grains are accumulated against the bottom surface of the piston. We call this accumulation "coffee grounds" (in black in figure 1). The piston is (partly or fully depending t) saturated with drinkable coffee. At the end of the process, all the coffee particles are confined between the piston and the tank bottom and the drinkable coffee (free of coffee particles) can be served.

- 1 pt 1. What is the definition of the saturation of a porous media ?
- 1 pt 2. During the process, coffee particles accumulate under the piston. What is the relation between the coffee grounds thickness e , the volume fraction ϕ , the piston's position z , the tank cross-section S and the coffee grounds porosity ε ?
- 1 pt 3. We assume that the piston moves at a constant velocity U . For the porous media formed by the coffee grounds, does U represent the intrinsic or superficial average of the velocity ? Give the definition of these two averages and the relation between by using the same notations as during the lessons.
- 1 pt 4. Compute the evolution of the coffee grounds thickness with time $e(t)$.
- 0.25 pt 5. We assume that coffee particles in the coffee grounds form a random packing. What is the approximate value of the porosity ε ?
- 0.5 pt 6. Give the expression of the pore Reynolds Re_p number for water flowing in the coffee grounds. Assuming that $U = 1 \text{ cm/s}$, compute an estimation of Re_p .
- 1.25 pt 7. As the Reynolds number Re_p is higher than 10, what is the law which relates the pressure gradient to the superficial average of the velocity ? Write the mathematical expression of this law using the lecture's notations.
- 0.5 pt 8. Which model must we use to estimate the coffee grounds permeability k_c ?
- 1.5 pt 9. Give the expression of k_c as a function of ε , η , ρ_w , U , d_p and other useful coefficient(s).
- 1 pt 10. Plot the trend of $k_c(U)$. What is the physical origin of the decrease on permeability with U ?
- 1 pt 11. Compute the effective permeability of the {piston+coffee grounds} system as a function of k_c and k_p . You can use results from the lecture, the whole demonstration is not expected.
- 2 pts 12. From the previous questions, typical porous media-related laws and standard hydrodynamics laws, show that the hydraulic resistance of the {piston+coffee grounds} system is :

$$R_h(U, t) = \frac{\eta}{S} \left(\frac{L}{k_p} + \frac{\phi U}{\eta(1-\varepsilon)} \left[\frac{180(1-\varepsilon)^2 \eta}{\varepsilon^3 d_p^2} + \frac{3\beta(1-\varepsilon)\rho_w U}{4\varepsilon^2 d_p} \right] t \right). \quad (1)$$

B An espresso ?

Another way of making an espresso is to inject pressurised water through the coffee particles. Coffee grounds are placed on a filter in a brewing chamber. Hot and pressurized water is applied above the coffee grounds with a pressure drop ΔP and flows through. We consider a saturated porous media made of one solid phase σ and on fluid phase α . Its porosity is denoted as ε . Figure 2 presents the system. The total height of coffee grounds is e considered as constant during the process. We consider a 1D problem in the vertical z direction. $z = 0$ corresponds to the top of the coffee grounds. During the dissolution process, grains

During the process, coffee particles are partly dissolved in the hot water.

We denote as P_α the pressure in the liquid phase, v_α the velocity of the fluid phase and C_α the dissolved coffee concentration in the liquid phase. We note $\langle \cdot \rangle^\alpha$ the intrinsic average (phase x) and $\langle \cdot \rangle$ the superficial one.

- 1 pt 1. We consider that the coffee grounds' permeability do not depend on flow velocity and is a scalar in this 1D problem. Write the Darcy's law. Define each term.
- 1 pt 2. Give the general volume-averaged equation of the advection-diffusion equation in a porous media. We will denote as \overline{D} the diffusion tensor.
- 0.5 pt 3. Summarize the different phenomenon taken into account in the diffusion tensor.
- 1 pt 4. Write the previous advection-diffusion equation applied to our 1D case. Simplify it by assuming a non compressible flow.

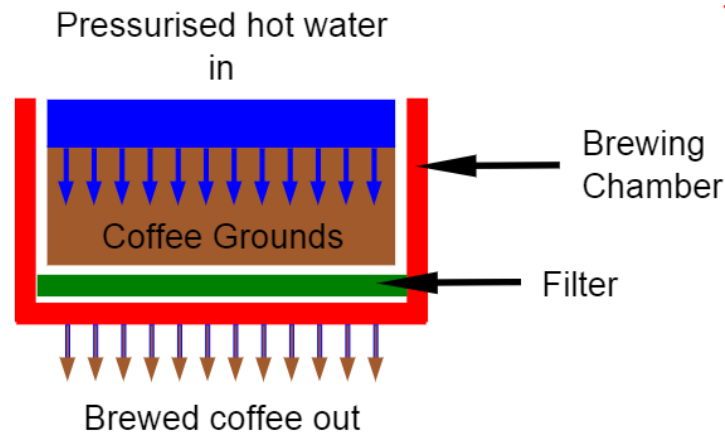


FIGURE 2 – Sketch of espresso coffee brewing (from Moroney *et al.*, Siam J. Appl. Math. (2016)).

- 1.5 pt 5. We consider a steady-state regime and a uniform permeability. Explain why the averaged velocity is uniform in the coffee grounds. What is the value of the averaged concentration at the outlet of the filter? What does it lack in the equation to avoid it? You will need to precise the value of $\langle C_\alpha \rangle^\alpha(z=0)$ and assume that $\frac{\partial \langle C_\alpha \rangle^\alpha}{\partial z}(z=0) = 0$.
- 3 pts 6. As detailed above, coffee particles are partly dissolved during the process. Propose a simple model to get the concentration profile in the coffee grounds and the final coffee concentration at the outlet. Detail your assumptions.

Exercise 2 : Article analysis

20 points

We propose to study an article entitled “The force required to operate the plunger on a French press” written by Fabian B. Wadsworth *et al.*. Before beginning the questions, have a quick overview of the article to see its structure and the main topic addressed. **Do not read the whole paper before beginning**, it will be a waste of time. Start by reading the abstract and the article’s structure. Your answers should be written with your own words, do not paraphrase.

Questions

A Generalities about the article

- 0.25 pt 1. Which is the year of publication of this article ?
- 0.25 pt 2. In which journal has it been published ?
- 2 pts 3. In few lines and without paraphrasing the text, summarize the objectives of the authors.

B Introduction

- 1 pt 1. Which processes are coupled during coffee extraction ?
- 0.5 pt 2. What kind of coffee has been extensively studied by researchers ?
- 0.5 pt 3. What is the title of the article on which the authors claim that French plunger is one of the most ecological method to brew coffee ?

C Material characterization and methods

- 0.5 pt 1. What is the mass of coffee recommended for a French coffee?
- 1 pt 2. What are the main differences in terms of methods between at-home and laboratory experiments?
- 0.5 pt 3. For the laboratory experiments, which physical quantity is kept constant, which one is measured?
- 1 pt 4. Detail the main specificities of a coffee grain in terms of size and structure.
- 0.25 pt 5. Pycnometry is a way to measure density. It can also help to measure porosity. Which type of pycnometry do the authors use?
- 1 pt 6. Which other type of porosimeter do you know? Explain how does it work in few sentences.

D Results and Analysis

- 0.25 pt 1. What is the typical time necessary to have a coffee pack of constant thickness (before pushing the plunger)?
- 1.5 pt 2. Detail computations and measurements used to get the coffee pack porosity to $\phi \approx 0.53$.
- 1 pt 3. In at-home experiments, explain why the displacement velocity of the plunger is constant and why the proportionality factor between $d(t)$ and t rises when the total coffee mass m decreases.
- 1 pt 4. How could you explain the transitory regime visible in figure 5?

E Coffee percolation scaling

- 1 pt 1. Which are the forces involved in the plunger displacement? Which of them are negligible?
- 1 pt 2. Detail the computations needed to get equation (3).
- 2 pts 3. In about ten lines, explain how is obtained the Darcy's law from the local Stokes law.
- 0.5 pt 4. What is the dependency between the hydrodynamic force required to squeeze the water through the pack of coffee and the total mass of coffee in the jug?

F The permeability of packed coffee

- 1 pt 1. Estimate the permeability from the green-circle curve of figure 4.
- 1 pt 2. Explain the sentence "Moreover, it is also a clear indication that the relevant velocity is the average velocity $\langle u \rangle$ of the water, and not the local velocities, which are both larger and highly variable in such a porous medium." in the light of what you have seen in class.
- 1 pt 3. Why don't the authors use the average radius in the Kozeny-Carman relation? What radius do they use?