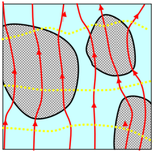
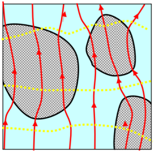


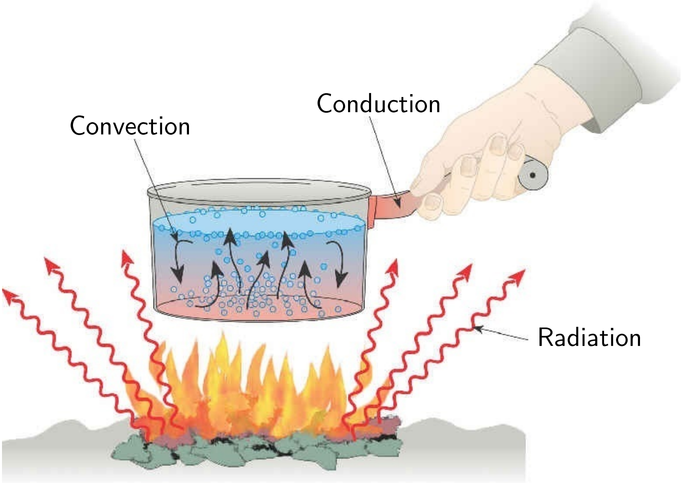
Thermal transfer in porous media



Olivier Liot Petit

Thermal transfer mechanisms

- In any media: several thermal transfer mechanisms



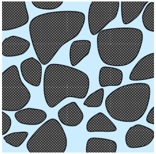
Adapted from A. Verschaere

- In porous media: co-existence of several phases with different thermal properties

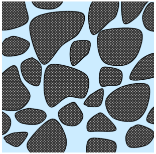
Motivations

Observations:

- Several way of thermal transport
- Different thermal properties ⇒ specific transfer at interfaces
- Transport of a scalar quantity through a moving phase



How to quantitatively describe thermal transfers in a porous media?



Dispersion and diffusion in porous media:

A. **Derivation of thermal conduction in a porous media**
B. **Models of convection in a porous media**

Objectives of this lecture’s section

Dispersion and diffusion in porous media

At the end of these lectures, you should be able to:

- cite and describe the three thermal transfer mechanisms in porous media
- summarize the conduction thermal transfer model
- interpret different models of equivalent conductivity

- differentiate natural and forced convection
- summarize the convection thermal transfer model
- define the Rayleigh and Nusselt numbers in porous media

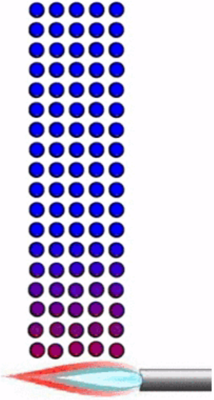
Thermal diffusion in homogeneous media: Fourier's law

- Diffusive, stationary flux
- Flux from high temperatures to low temperatures

Fourier's law

$$\overrightarrow{\phi} = -\overline{\overline{\lambda}} \cdot \overrightarrow{\mathrm{grad}}\, T$$

- $\overline{\overline{\lambda}}$: thermal conductivity tensor [W.m^{-1}.K^{-1}]
- $\overrightarrow{\phi}$: thermal flux density vector
- T: temperature [K]



- Conductivity can be affected by porous structure ⇒ anisotropic diffusion

Heat equation at microscopic scale: only conduction

- Liquid phase at rest, no heat source

Exercise: derivation of the heat equation

Heat equation at microscopic scale: only conduction

Take your device

Heat equation at microscopic scale: only conduction

- Liquid phase at rest, no heat source

Exercise: derivation of the heat equation

The heat equation for conduction is...

A. $\rho c_v \frac{\partial T}{\partial t} = \overline{\overline{\lambda}} \Delta T$

#QDLE#Q#AB*C##

B. $\rho c_v \frac{\partial T}{\partial t} = \mathrm{div}(\overline{\overline{\lambda}} \cdot \overrightarrow{\mathrm{grad}}\, T)$

C. $\rho c_v \frac{\partial T}{\partial t} = -\mathrm{div}(\overline{\overline{\lambda}} \cdot \overrightarrow{\mathrm{grad}}\, T)$

Heat equation at microscopic scale: only conduction

- Liquid phase at rest, no heat source

Exercise: derivation of the heat equation

The heat equation for conduction is...

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B. $\boldsymbol{\rho c_v \dfrac{\partial T}{\partial t}} = \mathrm{div}(\overline{\overline{\boldsymbol{\lambda}}} \cdot \overrightarrow{\mathrm{grad}} \boldsymbol{T})$

C. $\rho c_v \dfrac{\partial T}{\partial t} = -\mathrm{div}(\overline{\overline{\lambda}} \cdot \overrightarrow{\mathrm{grad}}\, T)$

- ρ: density
- T: temperature
- c_v: constant-volume specific heat

- $\overline{\overline{\lambda}}$ is not necessary scalar or uniform (anisotropic or inhomogeneous media)
- Valid for both phases while at rest

Homogenization of heat equations in a porous media

Local equations and boundary conditions

- $(\rho c_v)_\sigma \dfrac{\partial T_\sigma}{\partial t} = \mathrm{div}(\overline{\overline{\lambda_\sigma}} \cdot \overrightarrow{\mathrm{grad}}\, T_\sigma)$
- $(\rho c_v)_\alpha \dfrac{\partial T_\alpha}{\partial t} = \mathrm{div}(\overline{\overline{\lambda_\alpha}} \cdot \overrightarrow{\mathrm{grad}}\, T_\alpha)$
- $T_\sigma = T_\alpha \text{ on } S_{\sigma\alpha}$
- $\overline{\overline{\lambda_\sigma}} \cdot \overrightarrow{\mathrm{grad}}\, T_\sigma \cdot \overrightarrow{n}_{\sigma\alpha} = \overline{\overline{\lambda_\alpha}} \cdot \overrightarrow{\mathrm{grad}}\, T_\alpha \cdot \overrightarrow{n}_{\sigma\alpha}$

Remind – volume averages:

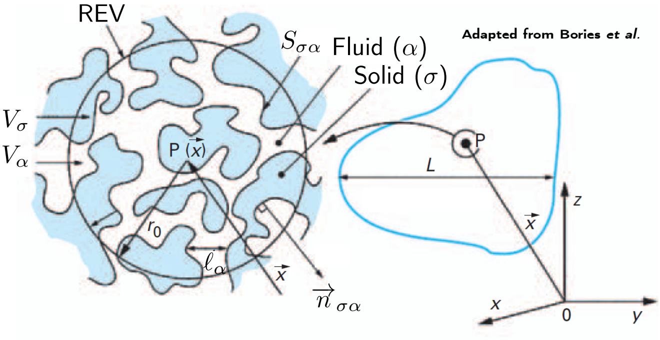
- Surface average: $\langle \Phi_\alpha \rangle = \dfrac{1}{V} \int_{V_\alpha} \Phi_\alpha dV$
- Intrinsic average: $\langle \Phi_\alpha \rangle^\alpha = \dfrac{1}{V_\alpha} \int_{V_\alpha} \Phi_\alpha dV$

Remind – volume averaging theorems:

- Gradient: $\langle \overrightarrow{\mathrm{grad}} \Phi_\alpha \rangle = \overrightarrow{\mathrm{grad}} \langle \Phi_\alpha \rangle + \dfrac{1}{V} \int_{S_{\alpha\sigma}} \Phi_\alpha \overrightarrow{n}_{\alpha\sigma} dS$
- Divergence: $\langle \mathrm{div} \overrightarrow{\Phi_\alpha} \rangle = \mathrm{div} \langle \overrightarrow{\Phi_\alpha} \rangle + \dfrac{1}{V} \int_{S_{\alpha\sigma}} \overrightarrow{\Phi_\alpha} \cdot \overrightarrow{n}_{\alpha\sigma} dS$
- Time-variation: $\langle \dfrac{\partial \Phi_\alpha}{\partial t} \rangle = \dfrac{\partial \langle \Phi_\alpha \rangle}{\partial t} - \dfrac{1}{V} \int_{S_{\alpha\sigma}} \Phi_\alpha \overrightarrow{W} \cdot \overrightarrow{n}_{\alpha\sigma} dS$
  - $\overrightarrow{n}_{\alpha\sigma}$: normal vector to σ and α interface
  - $S_{\alpha\sigma}$: σ and α interface
  - $\overrightarrow{W}$: displacement velocity vector of interface $S_{\alpha\sigma}$

Homogenization of heat equations in a porous media

- Rigid porous structure and only one non-solid phase $\overrightarrow{W} = \overrightarrow{0}$



Adapted from Bories *et al.*

ⓐ **microstructure** ⓑ **macrostructure**

Volume averaged heat equation

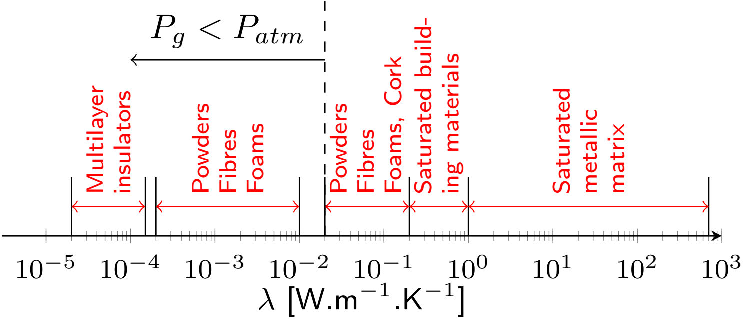
$$(\rho c_v)^* \frac{\partial T}{\partial t} = \mathrm{div}(\overline{\overline{\lambda}} \cdot \overrightarrow{\mathrm{grad}}\, T)$$

- Effective calorific capacity: $(\rho c_v)^* = \epsilon(\rho c_v)_\alpha + (1-\epsilon)(\rho c_v)_\sigma$

- Effective conductivity: $\overline{\overline{\lambda}} = (\epsilon \overline{\overline{\lambda_\alpha}} + (1-\epsilon)\overline{\overline{\lambda_\sigma}}) + \dfrac{\overline{\overline{\lambda_\alpha}} - \overline{\overline{\lambda_\sigma}}}{V} \cdot \int_{S_{\alpha\sigma}} \vec{b} \cdot \vec{n}_{\alpha\sigma}\, dS$

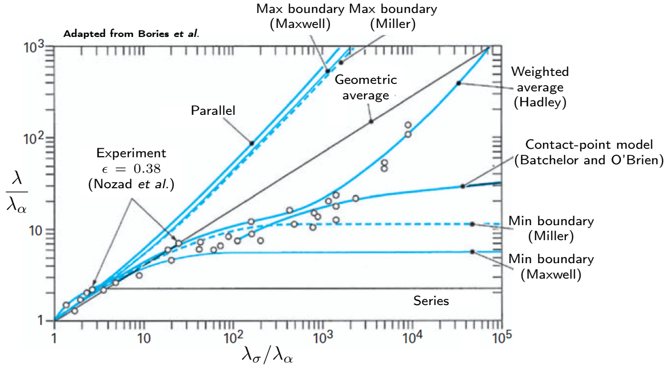
Effective thermal conductivity

- Some orders of magnitude



Effective thermal conductivity

- Several models (Maxwell, Miller, Hadley, Batchelor-O'Brien, ...)



Adapted from Bories *et al.*

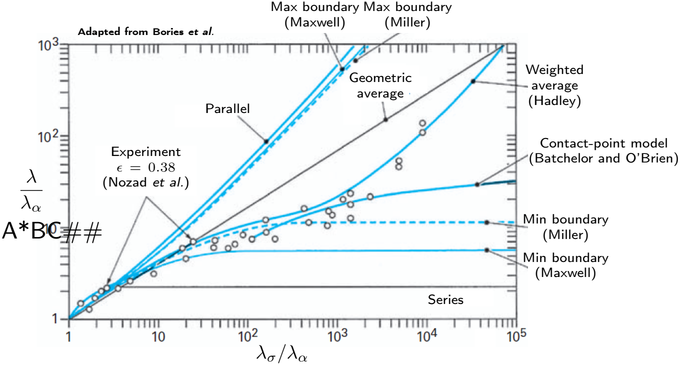
Exercise: derivation of the effective conductivity for simple arrangements

Effective thermal conductivity

Take your device

Effective thermal conductivity

- Several models (Maxwell, Miller, Hadley, Batchelor-O'Brien, ...)



#QDLE#Q#A*BC##

Exercise: derivation of the effective conductivity for simple arrangements

The effective conductivity for the parallel arrangement is...

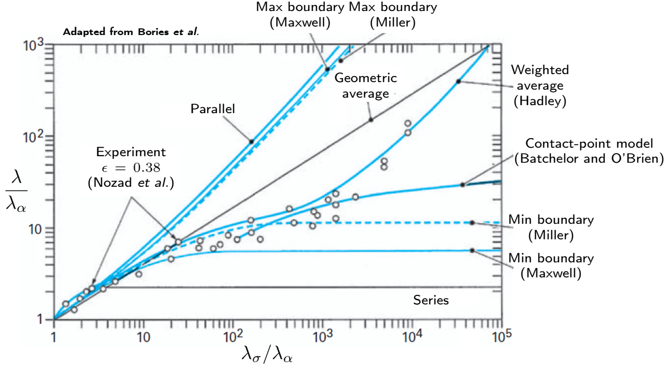
A. $\lambda = \lambda_\alpha \left(\epsilon + (1-\epsilon)\frac{\lambda_\sigma}{\lambda_\alpha}\right)$

B. $\lambda = \lambda_\alpha \left(\epsilon + (1-\epsilon)\frac{\lambda_\alpha}{\lambda_\sigma}\right)$

C. $\lambda = \lambda_\alpha \left(\epsilon + \frac{\lambda_\sigma}{\lambda_\alpha}\right)$

Effective thermal conductivity

> Several models (Maxwell, Miller, Hadley, Batchelor-O'Brien, ...)



Adapted from Bories *et al.*

Exercise: derivation of the effective conductivity for simple arrangements

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B. $$\lambda = \lambda_\alpha \left(\epsilon + (1-\epsilon)\frac{\lambda_\alpha}{\lambda_\sigma}\right)$$

C. $$\lambda = \lambda_\alpha \left(\epsilon + \frac{\lambda_\sigma}{\lambda_\alpha}\right)$$

Convection

- Fluid is moving ⇒ additional transport of heat
- Two kind of convections:

  - Natural convection
  - Forced convection

- Natural: driven only by density differences
- Forced: external wind imposed

Convection in porous media

- Hypothesis: local thermal equilibrium: $T = \langle T_\sigma \rangle^\sigma = \langle T_\alpha \rangle^\alpha$
- Darcy's law valid, no inertia, volume averaging

Volume averaged heat equation with convective part

$$(\rho c_v)^* \frac{\partial T}{\partial t} + (\rho c_v)_\alpha \langle \overrightarrow{v} \rangle \cdot \overrightarrow{\mathrm{grad}}\, T = \mathrm{div}\left[(\overline{\overline{\lambda}} + \overline{\overline{\lambda_d}}) \cdot \overrightarrow{\mathrm{grad}}\, T \right]$$

$$\langle \overrightarrow{v}_\alpha \rangle = -\frac{\overline{\overline{k}}}{\eta} (\overrightarrow{\mathrm{grad}} \langle P_\alpha \rangle^\alpha - \rho_\alpha \overrightarrow{g}) \qquad \text{and} \qquad \epsilon \frac{\partial \rho_\alpha}{\partial t} + \mathrm{div}(\rho_\alpha \langle \overrightarrow{v}_\alpha \rangle) = 0$$

- $\overline{\overline{\lambda_d}}$: effective thermal conductivity tensor due to dispersion

Diffusivity tensor

$$\overline{\overline{D}} = \frac{\overline{\overline{\lambda}} + \overline{\overline{\lambda_d}}}{(\rho c_v)_\alpha} = \frac{\overline{\overline{\lambda}}}{(\rho c_v)_\alpha} + \overline{\overline{D_d}}$$

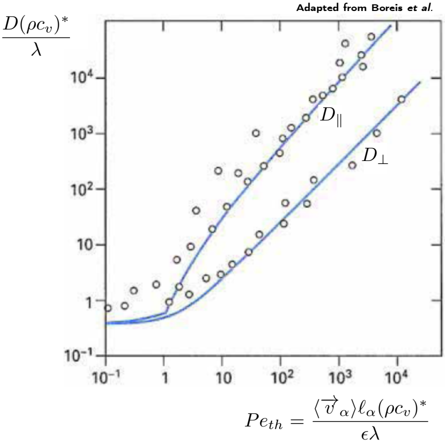
- Two components: longitudinal ($D_\parallel$) and transverse ($D_\perp$)

Competition convection/conduction

Thermal Péclet number

$$Pe_{th} = \frac{\text{convection}}{\text{conduction}} = \frac{\langle \overrightarrow{v}_\alpha \rangle \ell_\alpha (\rho c_v)^*}{\epsilon \lambda}$$

- Analog to Péclet number for dispersion
- If only diffusion, $D = \dfrac{\lambda}{(\rho c_v)^*}$



Adapted from Boreis *et al.*

Natural convection in porous media

- Dependence of ρ_α with T: $\rho_\alpha(T) = \rho_\alpha(T_0)(1 - \beta(T - T_0))$
  - β: thermal expansivity coefficient of the fluid

Exercise: dimensionless numbers in natural convection

Natural convection in porous media

Take your device

Natural convection in porous media

- Dependence of ρ_α with T: $\rho_\alpha(T) = \rho_\alpha(T_0)(1 - \beta(T - T_0))$
  - β: thermal expansivity coefficient of the fluid

Exercise: dimensionless numbers in natural convection

The Rayleihg number expression is...

A. $Ra = \frac{gdk(T_1 - T_2)}{\beta D\nu}$

#QDLE#Q#ABC*##

B. $Ra = \frac{\beta g d^3 k(T_1 - T_2)}{D\nu}$

C. $Ra = \frac{\beta g d k(T_1 - T_2)}{D\nu}$

Natural convection in porous media

- Dependence of ρ_α with T: $\rho_\alpha(T) = \rho_\alpha(T_0)(1 - \beta(T - T_0))$
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Exercise: dimensionless numbers in natural convection

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A. $Ra = \dfrac{gdk(T_1 - T_2)}{\beta D\nu}$

B. $Ra = \dfrac{\beta g d^3 k(T_1 - T_2)}{D\nu}$

C. $\boldsymbol{Ra = \dfrac{\beta g d k(T_1 - T_2)}{D\nu}}$

- Rayleigh number: comparison between convection “engines” and “breaks”
- “Prandtl” number: $Pr = \dfrac{\nu d^2}{kD}$

Thermal flux;

- Nusselt number: $Nu = \dfrac{\text{total heat flux}}{\text{conductive heat flux}}$

Natural convection in porous media

- Heat flux versus thermal forcing