MF2E department
3ème - Master FEIP
Toulouse INP – ENSEEIHT

Transfers in Porous Media
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Dispersion and diffusion in porous media

Exercises

1 Advection-dispersion equation

Let us consider a physical quantity (concentration, temperature, ...) held by a fluid. Two mechanisms can locally affect this quantity $C(x, y, z, t)$: molecular/particle diffusion and convection/advection due to the flow. We consider a control volume as the one presented figure 1. Advection and diffusion are supposed to be linearly independent.

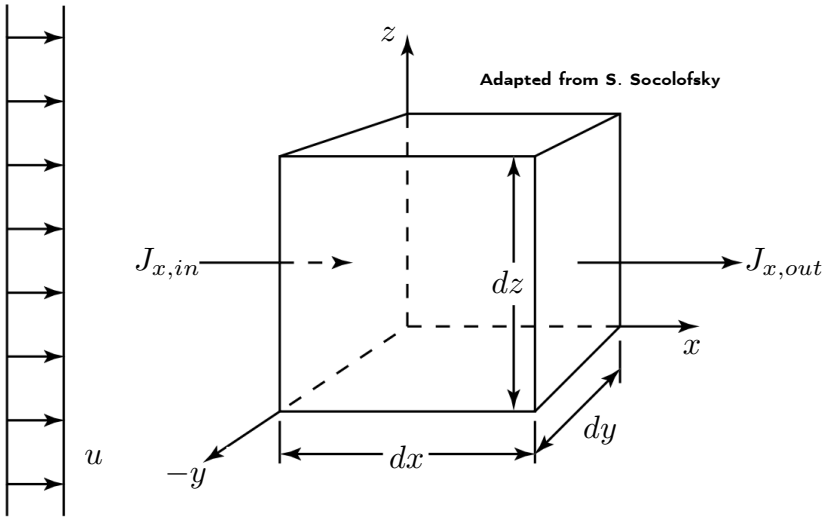

Figure 1: Sketch of a control volume with crossflow

1. Remind the Fick's law.

2. Derive the expression of the entering flux on the x face $J_{x,in}$.

3. Same question for $J_{x,out}$.

4. Assuming that dx, dy, and dz are very small, write the total flux J_x in the x direction.

5. Generalize to a 3-D problem (velocity $\overrightarrow{v} = (u, v, w)$) and give the total flux J.

6. To what is equal this flux under conservation assumption (no source)? Write the general advection-dispersion equation.

7. Assuming an uniform diffusivity coefficient, and an incompressible flow, simplify the advection-dispersion equation

2 2-D dispersion in a porous media

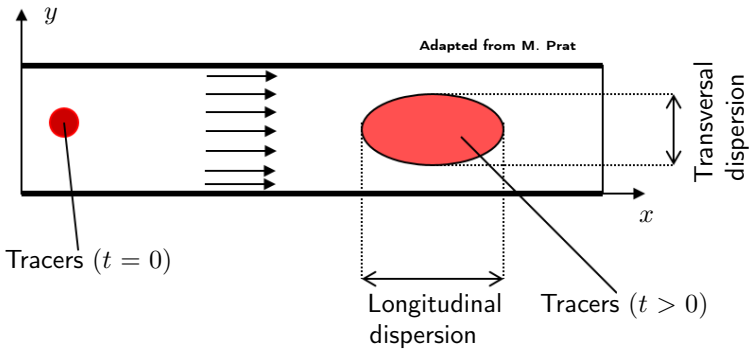

Figure 2: Flow and tracers in a porous media.

We remind the advection-dispersion equation in a porous media:

$$R\epsilon\frac{\partial\langle C\rangle^\alpha}{\partial t} = \epsilon\text{div}(\overline{\overline{D}}\cdot\overrightarrow{\text{grad}}\langle C\rangle^\alpha) - \text{div}(\langle\overrightarrow{v}\rangle\langle C\rangle^\alpha) \text{ with } R = 1 + K. \quad (1)$$

1. We consider no sorption. What is the value of R?

2. We denote as $D_{\|}$ the longitudinal part of the diffusivity tensor and $D_\perp$ its transverse component. Given the 2-D problem presented figure 2, write the advection-diffusion equation in this specific case.

3. If the typical pore dimension is ℓ_α, what is the Péclet number with the filtration velocity (or Darcy velocity)?