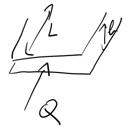
Permeability of a packed bed

1) Hele Shaw : $\langle v \rangle = \dfrac{h^2 \Delta P}{12 \eta L}$



Darcy : $\langle \vec{v} \rangle = -\dfrac{K}{\eta} \overrightarrow{\text{grad}} \langle P \rangle^{\alpha} \rightarrow \dfrac{\Delta P}{L}$

$\Rightarrow \boxed{K = \dfrac{h^2}{12}}$

2) $\varepsilon = \dfrac{V_{pores}}{V_{tot}} = \dfrac{V_{tot} - V_{particles}}{V_{tot}} = 1 - n \dfrac{\pi D^3}{6} \times \dfrac{1}{V}$

$a = \dfrac{S_{particles}}{V} = \dfrac{n \pi D^2}{V}$

$\Rightarrow \varepsilon = 1 - \dfrac{aD}{6} \iff \boxed{a = (1 - \varepsilon) \dfrac{6}{D}}$

specific surface, porosity, diameter

3) $U = \underbrace{\dfrac{\varepsilon^3}{2 \tau^2 a^2}}_{k} \dfrac{\Delta P}{\eta L} \Rightarrow k = \dfrac{\varepsilon^3}{2 \tau^2 a^2}$

4) $k = \dfrac{\varepsilon^3}{2 \tau^2 (1-\varepsilon)^2 \frac{36}{D^2}} = \dfrac{D^2 \varepsilon^3}{72 \tau^2 (1-\varepsilon)^2}$??

5) 

$\tau = \dfrac{L'}{D} = \dfrac{\pi D / 2}{D} = \dfrac{\pi}{2}$.

6) Kozeny - Carman relation.

$k = \dfrac{\varepsilon^3 D^2}{18 \pi^2 (1-\varepsilon)^2} \simeq \dfrac{\varepsilon^3 D^2}{180 (1-\varepsilon)^2}$

Kozeny - Carman factor

depends on the real beads spatial organization.

Inertia : Darcy - Forchheimer's law.

$\vec{v} \cdot \overrightarrow{\text{grad}}\, \vec{v} \rightarrow$ inertia term

Homogenization

$\hookrightarrow \overrightarrow{\text{grad}} \langle P \rangle^{\alpha} = -\eta \bar{\bar{K}}^{-1} \langle \vec{v} \rangle - \rho \beta \langle \vec{v} \rangle^2$

density

$\rightarrow \beta$ depends only on the porous media.

$\rightarrow \langle \vec{v} \rangle = -\overrightarrow{\text{grad}} \langle P \rangle^{\alpha} \dfrac{\bar{\bar{K}}}{\eta} - \rho \beta \|\bar{\bar{K}} \vec{v}\| \langle \vec{v} \rangle$

$\langle \vec{v} \rangle (1 + \rho \beta \|\vec{v}\|) = -\overrightarrow{\text{grad}} \langle P \rangle^{\alpha} \dfrac{\bar{\bar{K}}}{\eta}$

$\langle \vec{v} \rangle = -\dfrac{\bar{\bar{K}}_{eff}}{\eta} \overrightarrow{\text{grad}} \langle P \rangle^{\alpha}$

with $\bar{\bar{K}}_{eff} = \dfrac{\bar{\bar{K}}}{1 - \varepsilon \beta \sqrt{\bar{\bar{K}}}\, Re_p^*}$

where $Re_p^* = \dfrac{\rho \sqrt{\bar{\bar{K}}}\, \|\bar{\bar{K}} \vec{v}\|}{\eta \varepsilon}$

$\bar{\bar{K}}_{eff} = f(\|\bar{\bar{K}} \vec{v}\|)$

Ergun's law

$\hookrightarrow$ Packing bed

n cylinders (length L, diameter D)

$\Rightarrow$ Poiseuille $\Rightarrow \dfrac{\Delta P}{L} = \dfrac{32 \eta L \langle \vec{v} \rangle}{D^2}$

For a laminar flow, $\dfrac{\Delta P}{L} = 180 \dfrac{(1-\varepsilon)^2 \eta U}{\varepsilon^3 D_p^2}$

average velocity through the PM

particle diameter.

Dimensional analysis

$[\Delta P] = J \cdot m^{-3} = kg\, m^{-1} s^{-2}$

ΔP must be proportional to kinetic energy $/m^3$

$\Rightarrow \Delta P \propto L \, \rho \langle \vec{v} \rangle^2 \times \dfrac{1}{2} \times \dfrac{1}{D}$

$m \quad kg \cdot m^{-3} \quad m^2 s^{-2} \qquad m \quad = kg\, m^{-1} s^{-2}$

$\Rightarrow \dfrac{\Delta P}{L} = \dfrac{\rho}{2D} \langle \vec{v} \rangle^2$ contribution of inertia.

Moreover, $\langle \vec{v} \rangle = \varepsilon \langle \vec{v} \rangle^{\alpha}$

$U = \varepsilon \langle \vec{v} \rangle^{\alpha}$

Furthermore $\varepsilon = \dfrac{n}{V} L \pi \dfrac{D^2}{4}$

$a = \dfrac{n \pi D L}{V} \Rightarrow \varepsilon = a \dfrac{D}{4}$ cylinders.

and for a particle bed: $\varepsilon = 1 - \dfrac{a D_p}{6}$

Now we consider that PM 1 $\rightarrow$ assembly of cylinders

PM 2 $\rightarrow$ Packing bed

they have the same ε and a.

$\dfrac{\Delta P}{L} = \dfrac{\rho}{2D} \langle \vec{v} \rangle^{\alpha 2} = \dfrac{a \rho}{8 \varepsilon} \langle \vec{v} \rangle^{\alpha 2}$

We have the same specific surface between PM 1 and PM 2

$\Rightarrow \dfrac{\Delta P}{L} = \dfrac{3 \rho (1-\varepsilon)}{4 D_p \varepsilon^3} U^2$

$\hookrightarrow$ contribution of inertia to permeability.

$\dfrac{\Delta P}{L} = \underbrace{\dfrac{(1-\varepsilon)^2 \eta}{\varepsilon^3 D_p^2} U}_{\text{laminar}} + \beta \underbrace{\dfrac{3(1-\varepsilon)}{4 \varepsilon^3} \dfrac{\rho}{D_p} U^2}_{\text{inertia}}$

Ergun's correction

$\hookrightarrow$ tortuosity