MF2E DEPARTMENT
3ème année - Master FEIP
Toulouse INP – ENSEEIHT

TRANSFERS IN POROUS MEDIA
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Hydroynamic transport in a porous media

Exercises

1 Velocity through a cylinder assembly

Let us assume a system made of n identical cylinders (diameter D, length L). The cylinders are parallel and the porosity of this porous media is denoted as ϵ. The total porous media volume is V. A pressure drop ΔP is applied to the fluid filling the cylinders (viscosity η).

The mean fluid velocity through on cylinder is $\bar{v} = \dfrac{D^2 \Delta P}{32 \eta L}$.

1. Give the expression of the flow rate q through one cylinder.
2. We denote as S the total section of the porous media and U, the **superficial** average velocity through the porous media. Give the relation of the total flow rate Q as a function of U and S then n and q.
3. Deduce the expression U as a function of the problem parameters.
4. Give the relation between the porosity and the number of cylinders.
5. Compute the specific surface a of the porous media as a function of its volume V.
6. Express the mean velocity U as a function of the cylinders diameter, then as a function of the specific surface and the porosity.
7. We assume now that the cylinders have a tortuosity τ and that L is the global length of the porous media. How the expression of U is modified?

2 Derivation of the Darcy's law: first steps

The aim of this exercise is to derive the first steps of the homogenization of Stokes flow in a porous media using the volume-averaging method.

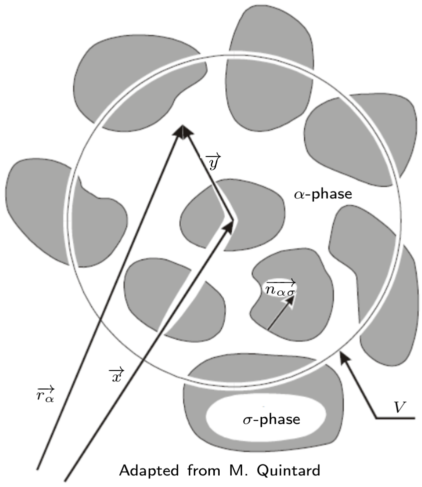


Figure 1: REV representation

We neglect gravity. The flow is considered as laminar and stationnary. A pressure drop is applied through the porous media to a fluid (viscosity η). Pressure is denoted as P and velocity as $\vec{v}$.

1. Give momentum and continuity equations.

2. Give the local boundary condition at the surface between solid and liquid phases $S_{\alpha\sigma}$.

Furthermore, we assume that the macroscopic boundary condition, at the surface between liquid phase and porous media surrounding environment is: $\vec{v} = \vec{f}(\vec{r}, t)$ at $S_{\alpha e}$.

3. Apply the appropriate averaging theorem to the continuity equation. What is the consequence on the intrinsic average velocity?

4. Apply the appropriate averaging theorem to the pressure term in the momentum equation.

5. We define $\widetilde{P}$ as the spatial fluctuations of pressure. Using the interfacial integral expression, simplify the relation obtained at the previous question.

3 Kozeny-Carman relation for the permeability

1. Compute the permeability of a Hele-Shaw cell whose plates are separated by a distance b. We remind that the average velocity in a Hele-Shaw cell is given by $\bar{v} = \dfrac{b^2 \Delta P}{12 \eta L}$.

2. We consider a packing of n spheric particles with porosity ϵ and volume V. Derive the relation between porosity and specific surface a.

3. We have seen earlier that the average velocity of fluid flowing through an assembly of tortuous parallel cylinders can be written as:

$$U = \frac{\epsilon^3}{2\tau^2 a^2} \frac{\Delta P}{\eta L}, \tag{1}$$

   where τ is the tortuosity, ΔP the pressure drop through the porous media of length L. What is the permeability k in this case?

4. We can consider that a packing bed of particles is an assembly of tortuous cylinders. In this case, give the expression of the permeability.

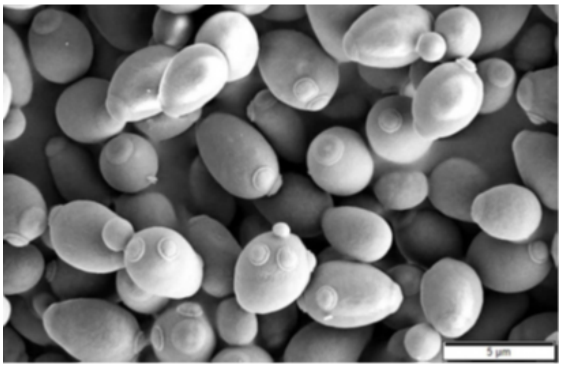
5. We need to estimate the tortuosity. We remind that we consider spherical particles. Show that we can estimate the tortuosity as $\tau = \pi/2$ by considering a fluid flowing close to a particle.

6. Conclude by giving the expression of the permeability for a packing bed of spherical particles (Kozeny-Carman relation).

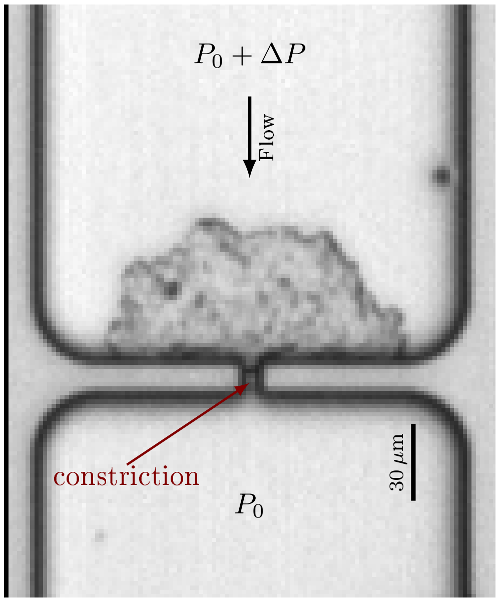
4 Hydrodynamic resistance of a yeast clog

This exercise aims at studying the hydrodynamic resistance of a yeast clog. Is is adapted form the 2022-2023 examination.

Yeasts are unicellular microorganisms widely present in nature and food industry. For example, baker's yeasts *Saccharomyces cerevisiae* are used to make both bread and beer. It is also a model organism used in a lots of biophysics studies. Their typical diameter is approximately 4.5 μm. Figure 2a show a micrography of yeasts *S. cerevisiae*.



(a) Baker's yeasts observed using scanning electron microscopy.



(b) Yeasts assembly trapped by a microfluidic constriction. A pressure drop ΔP is applied and the suspension flows from top to bottom.

Figure 2

Some very recent laboratory experiments consist in generating assembly of yeasts by trapping them in a microfluidic device in order to understand how does it react to various physico-chemical stress. Figure 2b shows a micrography of a yeast assembly during its construction. A suspension of yeasts is flowed in a microfluidic device using a pressure difference ΔP between the inlet and the outlet of the device. Its kinematic viscosity is $\nu = 2 \times 10^{-6}\,\mathrm{m^2/s}$ and its dynamic viscosity is $\mu = 2 \times 10^{-3}$ Pa.s. The microfluidic device is $h = 6\,\mu$m deep and $W = 140\,\mu$m wide. A constriction of $w = 6\,\mu$m traps the yeasts leading to the construction of a **quasi-2D** assembly of yeasts as you can see in figure 3. This assembly is considered as a porous media of porosity $\varepsilon = 0.1$.

1. As a first approximation, yeasts can be considered as spherical objects with a diameter of $d_p = 4.5 \pm 0.5\,\mu$m. Give the relation that we demonstrated during the lecture which relates the permeability k to the porosity ε and d_p. What is its name ?

2. The constriction is $6\,\mu$m deep and $6\,\mu$m wide. The inlet flow rate before yeast accumulation reaches around 300 nL/min for $\Delta P = 400$ mbar. Compute the order of magnitude of the Reynolds

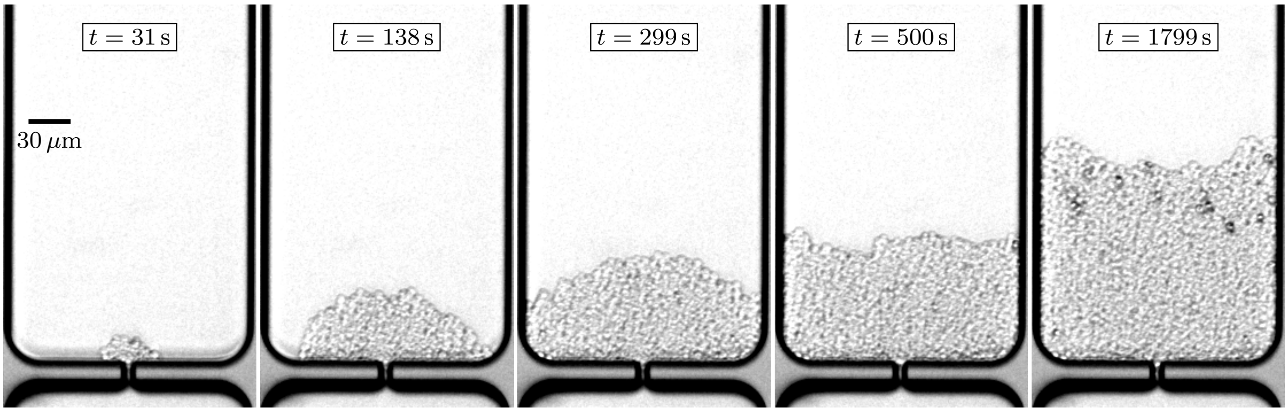


Figure 3: Sequence of pictures showing different steps of yeast assembly construction under flow.

number.

3. After few seconds, a yeast assembly appears at the constriction. It can be considered as a porous media. Based on the Reynols number value, write and give the name of the relation between fluid velocity and pressure gradient through this assembly using the notations proposed in the lecture.

4. Remind the hypotheses necessary to get this relation.

5. Give the definition of both intrinsic average and superficial average. What is the relation between these two averages ?

6. Summarize, without formula, the volume-averaging method used to derive the Darcy's law.

7. What should be the relation if inertia was not negligible (name and formula) ?

8. During the yeast accumulation, the assembly transits from a "cylindrical" regime to a "linear" regime. In the first regime, the yeast assembly can be considered as half a cylinder (height $h = 6\,\mu$m, radius R from the constriction). In the second regime, which is reached when $R \approx W$, it is a parallelepiped (height $h = 6\,\mu$m, width $W = 140\,\mu$m and length L from the constriction). Draw a sketch of each regime including dimensions.

9. We now consider that the permeability tensor can be reduced to a scalar k. For the first regime, give the relation between $\langle v \rangle$ and the flow rate Q then use it to integrate the Darcy law between $w/2$, the constriction half-width, and R. Show that :

$$Q(R) = \frac{\pi k h}{\mu} \frac{\Delta P}{\ln(2R/w)}. \tag{2}$$

10. For the second regime, we first consider the case where $L \gg W/2$, give the new relation between $\langle v \rangle$ and the flow rate Q then use it to integrate the Darcy law between W, the microfluidic inlet width, and L the assembly length. You should get a relation between ΔP, Q, L and other parameters of the problem.

11. We now consider that $L \gtrsim W$. In terms of hydraulic resistance, propose a way to model the clog?

12. We remind the generic formula relating flow rate and pressure drop: $\Delta P = R_h Q$. From the question 9., give the expression of the hydraulic resistance R_h^c of the cylindrical part of the clog for $R = W/2$.

13. Using the result of the question 10., show that the hydraulic resistance of the portion of the clog that extends in the area $z > W/2$ writes

$$R_h^l = \frac{\mu(L - W/2)}{Whk}. \tag{3}$$

14. What is the total hydraulic resistance of the clog in this configuration ?

15. We denote $P^\star$ as the pressure in the clog at a distance $W/2$ from the pore. Using an electrical analogy, compute the expression of $P^\star$ as a function of k, μ and some geometrical parameters.