



Gas Flow in Porous Media with Klinkenberg Effects

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Abstract. Gas flow in porous media differs from liquid flow because of the large gas compressibility and pressure-dependent effective permeability. The latter effect, named after Klinkenberg, may have significant impact on gas flow behavior, especially in low permeability media, but it has been ignored in most of the previous studies because of the mathematical difficulty in handling the additional nonlinear term in the gas flow governing equation. This paper presents a set of new analytical solutions developed for analyzing steady-state and transient gas flow through porous media including Klinkenberg effects. The analytical solutions are obtained using a new form of gas flow governing equation that incorporates the Klinkenberg effect. Additional analytical solutions for one-, two- and three-dimensional gas flow in porous media could be readily derived by the following solution procedures in this paper. Furthermore, the validity of the conventional assumption used for linearizing the gas flow equation has been examined. A generally applicable procedure has been developed for accurate evaluation of the analytical solutions which use a linearized diffusivity for transient gas flow. As application examples, the new analytical solutions have been used to verify numerical solutions, and to design new laboratory and field testing techniques to determine the Klinkenberg parameters. The proposed laboratory analysis method is also used to analyze data from steady-state flow tests of three core plugs from The Geysers geothermal field. We show that this new approach and the traditional method of Klinkenberg yield similar results of Klinkenberg constants for the laboratory tests; however, the new method allows one to analyze data from both transient and steady-state tests in various flow geometries.

Key words: gas flow, Klinkenberg effect, Klinkenberg constant, pneumatic analysis, unsaturated-zone flow, air venting, air permeability tests.

Notation

Roman Letters

a	lumped parameter in (3.5),
A	cross-section area, m^2 ,
b	Klinkenberg coefficient, Pa,
d	depth to well screen top, m,
\mathbf{g}	gravity vector, m/s^2 ,
h	formation thickness, m,
H	lumped parameter in (3.5),
K	lumped parameter in (3.5),
k_a	averaged gas permeability, m^2 ,
k_g	effective gas permeability, m^2 ,
k_∞	absolute permeability, m^2 ,
$k_{r,\infty}$	Klinkenberg permeability of low permeability layer in r -direction, m^2 ,
$k_{z,\infty}$	Klinkenberg permeability of low permeability layer in z -direction, m^2 ,
k'_∞	Klinkenberg permeability of low permeability layer, m^2 ,

l	depth to well screen bottom, m,
L	length of linear flow systems, or thickness of unsaturated zone, m,
L'	thickness of low permeability layer, m,
M_g	molecular weight of gas,
P	gas pressure, Pa,
P_0	gas pressure at inlet boundaries of linear flow systems, Pa,
P_b	gas pressure function (2.2), Pa,
P_i	initial gas pressure, Pa,
P_L	gas pressure at outlet boundaries of linear flow systems, Pa,
P_w	wellbore gas pressure, Pa,
\bar{P}	averaged (constant) gas pressure, Pa,
\bar{P}_b	averaged gas pressure function ($= \bar{P} + b$), Pa,
q_m	gas mass injection or pumping flux, kg/(s.m ²),
q_L^v	volumetric gas injection flux, m ³ /(s.m ²), measured at outlet pressure,
$Q_m, Q_{m,1}, Q_{m,2}$	gas mass injection or pumping rate, kg/s,
r	radial distance, m,
R	universal gas constant,
r_w	wellbore radius, m,
T	temperature, °C,
t	time, s,
\mathbf{v}	Darcy's velocity, m/s,
x	linear distance, m,
z	vertical distance, m.

Greek Letters

α	gas diffusivity (2.3),
β	compressibility factor,
ϕ	porosity,
Φ	gas pressure function, defined in (3.3), Pa ² ,
λ_n	defined in (3.6),
μ	viscosity, Pa s,
ρ	gas density, kg/m ³ ,

Subscripts

g	gas,
i	initial,
L	outlet at $x = L$,
m	mass,
0	inlet at $x = 0$,
w	well.

1. Introduction

Gas flow in porous media has recently received considerable attention because of its importance in the areas of pneumatic test analysis, contaminant transport and remediation in the unsaturated zone, and vadose zone hydrogeology. Quantitative analysis of gas flow and gas-phase transport is critical to these environmental protection and restoration projects. Therefore, analytical solutions and numerical models have been used extensively in these studies and applications.

One focus of current research in the fields of unsaturated-zone hydrology and soil physics is to develop economically feasible remediation schemes to clean up contamination in shallow aquifers. Typical contaminants in unsaturated zones are volatile organic chemicals (VOCs) and non-aqueous phase liquids (NAPLs) which have been spilled from leaking storage tanks or pipelines. Once these contaminants enter the subsurface, it is very difficult to remove them because of strong capillary and chemical forces between these contaminants and the soil particles, which is complicated by the heterogeneous nature of soils. Among currently used *in situ* remediation techniques, soil–vapor extraction and air sparging have proven to be very efficient and cost-effective methods for the removal of VOCs or NAPLs from unsaturated soils. The successful application of these techniques depends on a thorough understanding of gas flow dynamics and site conditions. As a result, many analytical solutions (Johnson *et al.*, 1990; McWhorter, 1990; Baehr and Hult, 1991; Shan *et al.*, 1992; Baehr and Joss, 1995, and Shan, 1995) and numerical models (Weeks, 1978; Wilson *et al.*, 1987; Baehr *et al.*, 1989; Mendoza and Frind, 1990; Pruess 1991; Falta *et al.*, 1992; Huyakorn *et al.*, 1994; Panday *et al.*, 1995) have been developed for analyzing gas flow in the unsaturated zone.

The systematic investigation of gas flow in porous media was pioneered in the petroleum industry for the development of natural gas reservoirs (Muskat, 1946). The use of gas flow models has been a standard technique in the petroleum industry for estimating gas permeability and other reservoir parameters in natural gas production (Dake, 1978; and Ikoku, 1984). There exists a considerable amount of studies on theory and application of isothermal flow of gases through porous media in the petroleum literature. The earliest attempt to solve gas flow problems used the method of successions of steady states proposed by Muskat (1946). Approximate analytical solutions (Katz *et al.*, 1959) were then obtained by linearizing the flow equation for an ideal gas to yield a diffusion-type equation. Such solutions were found to be of limited general use because of the assumption introduced to simplify the gas properties and the flow equation. The reasons are that, in general, gas flow in deep pressurized gas reservoirs does not follow the ideal gas law, and the variations of pressure around gas production wells are too large to use constant properties. It was not until the mid sixties that more reliable mathematical solutions were developed using a numerical method (Russell *et al.*, 1966) and introducing a real gas pseudo-pressure function (Al-Hussainy *et al.*, 1966).

In recent years, hydrologists and soil scientists have applied similar techniques to conduct soil characterization studies by pneumatic testing of air flow properties. Pneumatic test analysis has become an important methodology in determining formation properties of two-phase unsaturated-zone flow in a proposed repository of high-level radionuclear waste (Ahlers *et al.*, 1995). Because the ideal gas law is a better approximation to the near surface air flow than in deep gas reservoirs and also the pressure changes in the unsaturated zone are generally small, the simple linearization using an ambient, averaged gas pressure in evaluating the gas diffusivity term in the flow equation may be suitable for many unsaturated-zone applications.

While the numerical models developed can be used to perform rigorous modeling studies of gas flow under complex conditions, analytical solutions continue to provide a simple tool to determine gas flow properties. Despite the progress made so far in our understanding of porous medium gas flow, one important aspect, the Klinkenberg effect (Klinkenberg, 1941), has been ignored in most studies. Even though efforts have been made to estimate errors introduced by neglecting the Klinkenberg effect (Baehr and Hult, 1991), only a few studies address this phenomenon explicitly. Gas flows in porous media differently from liquid; first, because gas is highly compressible, and second, because of the Klinkenberg effect. The Klinkenberg effect may have significant impact on gas flow behavior, especially in low permeability media. Some recent laboratory studies (Reda, 1987; Persoff and Hulen, 1996) concluded that the Klinkenberg effect is important in the low permeability formations studied and cannot be ignored.

According to Klinkenberg (1941), effective gas permeability at a finite pressure is given by

$$k_g = k_\infty \left(1 + \frac{b}{P} \right), \quad (1.1)$$

where k_∞ is the absolute, gas-phase permeability under very large gas-phase pressure at which condition the Klinkenberg effects are negligible; and b is the Klinkenberg factor, dependent on the pore structure of the medium and temperature for a given gas.

Physically, Klinkenberg effects are significant in any situation where the mean free path of gas molecules in porous media approaches the pore dimension, i.e. when significant molecular collisions are with the pore wall rather than with other gas molecules. Gas permeability is then enhanced by 'slip flow'. Therefore, it has been expected that Klinkenberg effect is the greatest in fine-grained, lower permeability porous media. Jones (1972) found that b generally decreases with increasing permeability according to

$$b \propto k_\infty^{-0.36}, \quad (1.2)$$

based on a study using 100 cores ranging in permeability from 0.01 to 1000 md. Typical values of b may be estimated as listed in Table I.

This paper presents a set of analytical solutions developed to analyze steady-state and transient gas flow through porous media with Klinkenberg effects. A new variable (pressure function) is used to simplify the gas flow governing differential equation with the Klinkenberg effect. In term of the new variable, the gas flow equation has the same form as that without including the Klinkenberg effect under the same linearization assumption. As a result, many one-, two- and three-dimensional gas flow solutions can be readily derived by analogy to non-Klinkenberg gas flow, slightly compressible single-phase liquid flow or heat conduction problems.

As examples of application, the analytical solutions have been used to verify the numerical solutions for simulating Klinkenberg effects and to provide linear

Table I. Typical values of the Klinkenberg factor, b

k_g (m ²)	b (Pa)
10^{-12}	3.95×10^3
10^{-15}	4.75×10^4
10^{-18}	7.60×10^5

correlations according to which laboratory data can be plotted to determine the values of k_∞ and b . To demonstrate the application of the proposed laboratory technique to determining the Klinkenberg parameters, steady-state, single-phase gas flow tests have been conducted using three core plugs of Graywacke from well NEGU-17 of The Geysers geothermal field in California. The gas permeability measurements are analyzed using the proposed method, and consistent results have been obtained for Klinkenberg coefficients, as compared with the traditional method.

2. Gas Flow Equation with Klinkenberg Effects

If a subsurface system is isothermal, the ideal gas law applies, and gravity effects are negligible, then gas flow in porous media is described by a mass balance equation (see Appendix A),

$$\nabla \cdot (\nabla P_b^2) = \frac{1}{\alpha} \frac{\partial P_b^2}{\partial t}, \quad (2.1)$$

where we use a new variable (Collins *et al.*, 1953), the pressure function:

$$P_b = P + b \quad (2.2)$$

and α is a gas diffusivity, defined as a function of gas pressure,

$$\alpha = \frac{k_\infty P_b}{\phi \mu}. \quad (2.3)$$

Equation (2.1) is identical to the gas flow governing equation which does not include the Klinkenberg effects with P_b being replaced by P .

In addition to Klinkenberg effects, porous media gas flow may be affected by turbulent or inertial effects (Tek *et al.*, 1962; Dranchuk *et al.*, 1968, 1969; Katz *et al.*, 1990; Lee *et al.*, 1987). However, significant turbulent flow usually occurs in formations with high permeability. By using Equation (1.2), the correlation of the turbulence factor given by Tek *et al.* (1962), and a modified Forchheimer equation, it can be shown that effects of turbulent flow can in general be ignored when Klinkenberg effects are significant. Therefore, turbulent effects on gas flow are not included in the following solutions and analyzes.

3. Analytical Solutions

The gas flow Equation (2.1) is a nonlinear partial differential equation with respect to P_b^2 because of the diffusivity term α , (2.3), which is a function of pressure. In general, the gas flow governing Equation (2.1) needs to be solved by a numerical method. However, it is possible to obtain certain analytical solutions as proven in the following flow conditions.

3.1. STEADY-STATE SOLUTIONS

Under steady-state flow conditions, Equation (2.1) becomes linear and many analytical solutions can be directly derived using solutions from corresponding slightly compressible fluid flow or heat conduction problems. Two examples are given in this section to demonstrate solution procedures. The first solution is needed in Section 5 for application, and the second has applicability to a field problem.

3.1.1. Linear Flow

Under one-dimensional, linear, horizontal and steady-state flow conditions, Equation (2.1) can be simplified as

$$\frac{\partial}{\partial x} \left(\frac{k_{\infty} \beta (P + b)}{\mu} \frac{\partial P}{\partial x} \right) = 0. \quad (3.1)$$

The boundary conditions are: at the inlet ($x = 0$), a constant mass injection rate q_m per unit cross-sectional area is imposed, and at the outlet ($x = L$), the gas pressure is kept constant. Then, a steady-state solution can be written as follows:

$$P(x) = -b + \sqrt{b^2 + P_L^2 + 2bP_L + 2q_m \mu (L - x) / k_{\infty} \beta}. \quad (3.2)$$

3.1.2. Two-Dimensional (r - z) Axisymmetric Flow

The second example will demonstrate how to derive a new analytical solution with Klinkenberg effects using existing non-Klinkenberg gas flow solutions. The 2-D (radial and vertical), steady-state flow problem was described in detail by Baehr and Joss (1995). When Klinkenberg effects are included, the flow equation becomes

$$k_{r,\infty} \frac{\partial^2 \Phi}{\partial r^2} + k_{r,\infty} \frac{1}{r} \frac{\partial \Phi}{\partial r} + k_{z,\infty} \frac{\partial^2 \Phi}{\partial z^2} = 0, \quad (3.3)$$

where $\Phi = (P + b)^2$, and $k_{r,\infty}$ and $k_{z,\infty}$ are the Klinkenberg permeabilities in r - and z - directions, respectively, which are different for an anisotropic system.

The problem concerns airflow to or from a partially penetrating well in an unsaturated zone that is separated from the atmosphere by a low-permeability, horizontal

layer on the top. At the interface ($z = 0$) between the unsaturated zone and the top low-permeability layer, the continuity in pressure and mass flux requires

$$k_{z,\infty} \frac{\partial \Phi}{\partial z} = \frac{k'_{\infty}}{L'} (\Phi - \Phi_{\text{atm}}) \quad \text{for } r > r_w, \quad z = 0, \quad (3.4)$$

where $\Phi_{\text{atm}} = (P_{\text{atm}} + b)^2$, k'_{∞} and L' are the Klinkenberg permeability and thickness of the top low-permeability layer. The other boundary conditions are the same as described by Baehr and Joss (1995), except that the boundary conditions are expressed in terms of Φ . Then, a steady-state solution for this problem can be derived for gas pressure distribution in the r - z system using the solution of Baehr and Joss (1995) as

$$\Phi(r, z) = \Phi_{\text{atm}} + K \left\{ \sum_{n=1}^{\infty} \alpha_n \cos \left[\frac{\lambda_n (L - z)}{L} \right] K_0 \left(\frac{\lambda_n r}{aL} \right) \right\}, \quad (3.5)$$

where

$$K = \frac{2H\mu Q_m aL}{\pi k_{r,\infty} (l - d)r_w \beta}, \quad a = \left(\frac{k_{r,\infty}}{k_{z,\infty}} \right)^{1/2}, \quad H = \frac{(k'_{\infty} L)}{(k_{z,\infty} L')},$$

and

$$\alpha_n = \frac{\sin[\lambda_n (L - d)/L] - \sin[\lambda_n (L - l)/L]}{\lambda_n^2 K_1(\lambda_n r_w / aL) (H + \sin^2 \lambda_n)}.$$

Here functions K_0 and K_1 are the zero-order and first-order modified Bessel functions of the second kind, respectively, and λ_n ($n = 1, 2, 3, \dots$) are the roots of the equation:

$$\tan(\lambda_n) = \frac{H}{\lambda_n}. \quad (3.6)$$

3.2. TRANSIENT SOLUTIONS

Equation (2.1) may be linearized using the conventional approach for transient gas flow analysis, i.e. set

$$\alpha = \frac{k_{\infty} P_b}{\phi \mu} \approx \frac{k_{\infty} \bar{P}_b}{\phi \mu}, \quad (3.7)$$

where $\bar{P}_b = \bar{P} + b$, is a function of average gas pressure, \bar{P} , and is treated as a constant. With the approximation of (3.7), Equation (2.1) becomes linear with respect to P_b^2 , and many analytical solutions can be obtained by analogy with heat conduction problems (Carslaw and Jaeger, 1959) or slightly compressible flow problems, as demonstrated in the following.

3.2.1. Flow in an Infinite Radial System with a Constant Line Source

For horizontal radial flow towards a well, approximated as a line source/sink, in an infinite, uniform, and horizontal formation, the gas flow Equation (2.1) may be expressed in terms of P_b^2 ,

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial P_b^2}{\partial r} \right) = \frac{1}{\alpha} \frac{\partial P_b^2}{\partial t} \quad (3.8)$$

with the uniform initial condition:

$$P_b^2 = (P_i + b)^2 = P_{bi}^2 \quad \text{for } t = 0, \quad t > 0, \quad (3.9)$$

where P_i is a constant gas pressure of the formation.

The well boundary condition proposed are as a line source/sink well:

$$\lim_{r \rightarrow 0} \frac{\pi k_{\infty} h r \beta}{\mu} \frac{\partial P_b^2}{\partial r} = Q_m, \quad (3.10)$$

where Q_m is a constant mass pumping or injection rate.

At the large distance from the well, the pressure is kept constant, i.e.

$$P_b^2 = (P_i + b)^2 = P_{bi}^2 \quad \text{for } r \rightarrow \infty. \quad (3.11)$$

With the linearization to the diffusivity α , (3.7), the problem of Equations (3.8)–(3.11) for the line source/sink case is identical to the Theis solution (Theis, 1935) in terms of P_b^2 . Then, the solution can be written directly as

$$P_b^2(r, t) = P_{bi}^2 - \frac{\mu Q_m}{2\pi k_{\infty} h \beta} \text{Ei} \left(-\frac{r^2}{4\alpha t} \right). \quad (3.12)$$

3.2.2. Flow in an Infinite Radial System with a Constant Wellbore Pressure

For the case of flow under the constant wellbore pressure (P_w) in an infinite radial system, the wellbore boundary condition becomes

$$P_b^2 = (P_w + b)^2 = P_{bw}^2 \quad \text{for } t > 0, \quad r = r_w. \quad (3.13)$$

Equations (3.8), (3.9), (3.11) and (3.13) for this case are identical to the heat conduction problem (Carslaw and Jaeger, 1959, p. 335) in terms of P_b^2 . The solution for pressure is

$$P_b^2(r, t) = P_{bw}^2 - \frac{2(P_{bw}^2 - P_{bi}^2)}{\pi} \times \int_0^{\infty} \exp(-\alpha u^2 t) \frac{J_0(ur)Y_0(ur_w) - J_0(ur_w)Y_0(ur)}{J_0^2(ur_w) + Y_0^2(ur_w)} \frac{du}{u}, \quad (3.14)$$

where J_0 and Y_0 are the Bessel functions of order 0 of the first and second kind, respectively.

4. Evaluation of Analytical Solutions

The steady-state solutions derived above are exact solutions, and can be directly applied to analyzing gas flow under steady-state flow conditions. However, the transient solutions of gas flow provided in Section 3 are approximate solutions because they assume a constant gas diffusivity, Equation (3.7), to linearize the gas flow Equation (2.1). Such solutions, though widely used in the analysis of transient gas flow in unsaturated zones (Weeks, 1978; and Shan, 1995), need to be further investigated for the validity of the linearization assumption and for the conditions under which these solutions apply. In the petroleum literature, it has been found that in many situations, the linearization assumption is inappropriate when applied to the flow of a real gas in reservoirs (Dake, 1978; Ikoku, 1984; Al-Hussainy *et al.*, 1966; and Russell *et al.*, 1966). This may be due to the high pressure in a gas reservoir. When applied to the near surface air flow analysis, the same linearization procedure may give reasonable accuracy for gas flow in unsaturated zones due to small (a few percent) surface atmospheric pressure changes (Kidder, 1957). Nevertheless, the applicability of such a linearization approximation to any particular problem should be critically examined.

The applicability of the linearized gas flow solutions to different situations depends mainly on how well an averaged formation pressure can be used to obtain a representative gas diffusivity term in (3.7) in the pressure disturbed zone. The conventional treatment, when Klinkenberg effects are ignored, is

$$\bar{P} \approx P_i, \quad (4.1)$$

where P_i is the initial, constant gas pressure of the system. This scheme may provide reasonable accuracy for certain pneumatic analysis (Shan, 1992) when the overall pressure changes or perturbations are small relative to initial pressure values of the system. However, using (4.1) to evaluate the diffusivity will introduce a large error when gas pressure changes are significant, such as in air sparging operations. We propose to use a history-dependent, averaged pressure within the pressure changed (disturbed) domain instead of a constant diffusivity, evaluated using (4.1). The history-dependent averaged pressure is defined as:

$$\bar{P} \approx \frac{\sum A_j P_j}{\sum A_j}, \quad (4.2)$$

where A_j is a controlled area at the geometric center of which the pressure was P_j at the immediate previous time when the solution was calculated. The summation, $\sum A_j$, is done over all A_j where pressure increases (or decreases) occurred at the previous time value. P_j is always evaluated analytically at point j , based on the previous estimated, constant diffusivity.

The reasoning for the proposed scheme is that the diffusivity of (3.7) may be better approximated when using an averaged, history-dependent pressure of (4.2). Otherwise, if (4.1) is used throughout in the solution in evaluating the diffusivity

term, it remains constant. This may introduce significant errors to the solution, in particular, at late times when pressures and their distributions in the system are very different from the initial condition.

To demonstrate the proposed scheme for better estimation of the nonlinear diffusivity term in the gas flow equations, we present the following comparison study using a numerical model. A numerical code for multiphase flow, TOUGH2 (Pruess, 1991), is used here to examine the approximate transient gas flow solution. The TOUGH2 code has been verified extensively for its accuracy in simulating gas flow in porous media (Pruess *et al.*, 1996). Verification examples for gas flow with the Klinkenberg effect are provided in the next section. The test problem concerns single-phase isothermal transient gas flow in a radially infinite system with constant gas mass injection rate through a line source.

The parameters used for this comparison study are: porosity $\phi = 0.3$; permeability coefficient $k_{\infty} = 1 \times 10^{-15} \text{ m}^2$; Klinkenberg coefficient $b = 4.75 \times 10^4 \text{ Pa}$; formation temperature $T = 25^{\circ}\text{C}$; compressibility factor $\beta = 1.18 \times 10^{-5} \text{ kg/Pa m}^3$; gas viscosity $\mu = 1.84 \times 10^{-5} \text{ Pa s}$; initial pressure $P_i = 10^5 \text{ Pa}$; and thickness of the radial system is 1 m. The well boundary condition is: air mass injection rate $Q_m = 1 \times 10^{-5}$ and $1 \times 10^{-4} \text{ kg/s}$.

Figure 1 presents the comparisons of the pressure profiles at 1 day calculated from the numerical (true) and analytical (approximate) solutions. At the lower injection rate of $Q_{m,1} = 1 \times 10^{-5} \text{ kg/s}$ for the gas flow problem, the pressure increase in the system is relatively small at 1 day, and the analytical solution using P_i for \bar{P} gives excellent

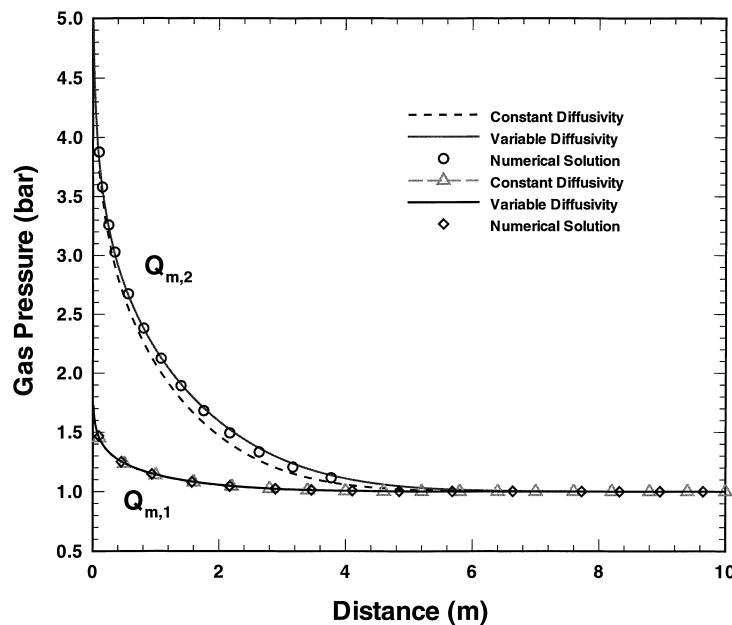


Figure 1. Comparison of gas pressure profiles in a radially infinite system at 1 day, calculated using the numerical and the analytical solutions.

with the numerical solution. However, as the injection rate increases ($Q_{m,2} = 1 \times 10^{-4} \text{ kg/s m}^2$), the gas pressure increases significantly. The analytical solution, with P_i as the averaged system pressure (Constant Diffusivity), gives poor accuracy, as shown in Figure 1. However under the same injection rate, the proposed scheme for evaluating the nonlinear diffusivity (Variable Diffusivity) using a history-dependent averaged pressure (4.2) results in excellent agreement with the numerical solution. Constant-diffusivity solutions give larger errors with larger injection rate in all the cases when compared with numerical solutions. Numerical tests for one-dimensional radial and linear flow problems indicate that the new scheme always results in more accurate solutions than the constant-diffusivity method, when compared with the numerical solutions (Wu *et al.*, 1996).

5. Application

In this section, several application examples will be given for the analytical solutions derived in Section 3. The application problems include: (1) checking the numerical scheme; (2) laboratory determination of Klinkenberg coefficients; (3) transient well tests; and (4) laboratory test analysis.

5.1. EXAMINATION OF NUMERICAL SCHEME

5.1.1. Steady-State Flow

This is to examine the accuracy of the TOUGH2 formulation in simulating porous medium gas flow with the Klinkenberg effect. The problem concerns steady-state gas flow across a linear rock column 10 m long. The system contains single-phase gas at isothermal condition, and a constant gas mass injection rate is imposed at the inlet of the column. The outlet end of the rock column is kept at a constant pressure. Eventually, the system will reach steady state.

The formation and Klinkenberg properties were selected from a laboratory study of the welded tuff at Yucca Mountain (Reda, 1987). The parameters used are: porosity $\phi = 0.3$; permeability $k_\infty = 5 \times 10^{-19} \text{ m}^2$; Klinkenberg coefficient $b = 7.6 \times 10^5 \text{ Pa}$; formation temperature $T = 25^\circ\text{C}$; and compressibility factor $\beta = 1.18 \times 10^{-5} \text{ kg/Pa m}^3$; gas viscosity $\mu = 1.84 \times 10^{-5} \text{ Pa s}$. The boundary conditions are: air mass injection rate $Q_m = 1 \times 10^{-6} \text{ kg/s}$; the outlet boundary pressure $P_L = 1 \times 10^5 \text{ Pa}$; and cross-area $A = 1 \text{ m}^2$.

A comparison of the pressure profile along the rock column from the TOUGH2 simulation and the exact, analytical solution (3.2) is shown in Figure 2, indicating that the TOUGH2 simulated pressure distribution is in excellent agreement with the analytical solution for this problem.

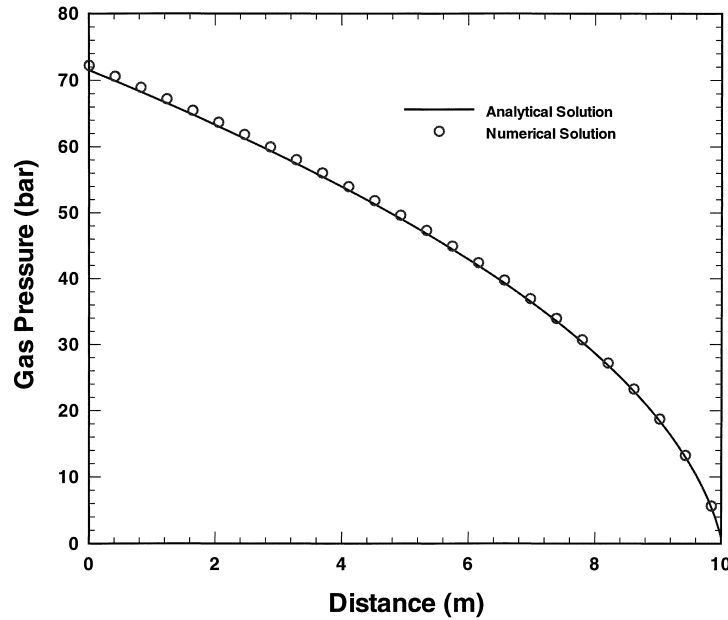


Figure 2. Comparison of the analytical and the numerical solutions for steady-state gas flow in a finite linear system.

5.1.2. Transient Flow

This is to examine the capability of the TOUGH2 formulation in simulating transient gas flow with the Klinkenberg effects. The problem concerns gas injection into a well in a large horizontal, uniform, and isothermal formation. A constant gas mass injection rate is imposed at the well, and the initial pressure is uniform throughout the formation.

The parameters used are porosity $\phi = 0.3$; permeability $k_{\infty} = 1 \times 10^{-15} \text{ m}^2$; Klinkenberg coefficient $b = 4.75 \times 10^4 \text{ Pa}$; The air mass injection rate $Q_m = 1 \times 10^{-6} \text{ kg/s}$; the initial formation pressure $P_1 = 1 \times 10^5 \text{ Pa}$; the wellbore radius, $r_w = 0.1 \text{ m}$; the formation thickness, $h = 1 \text{ m}$; and μ and T are the same as in the steady-state flow case above.

A comparison of the pressure profiles along the radial direction after ten days of injection from the TOUGH2 simulation and the analytical solution (3.12) is shown in Figure 3. Again, excellent agreement has been obtained for the transient flow problem.

5.2. LABORATORY DETERMINATION OF PERMEABILITY AND KLINKENBERG COEFFICIENT

The traditional method used in laboratory determination of the permeability and the Klinkenberg coefficient is using a plot of averaged gas permeability k_a vs. inverse

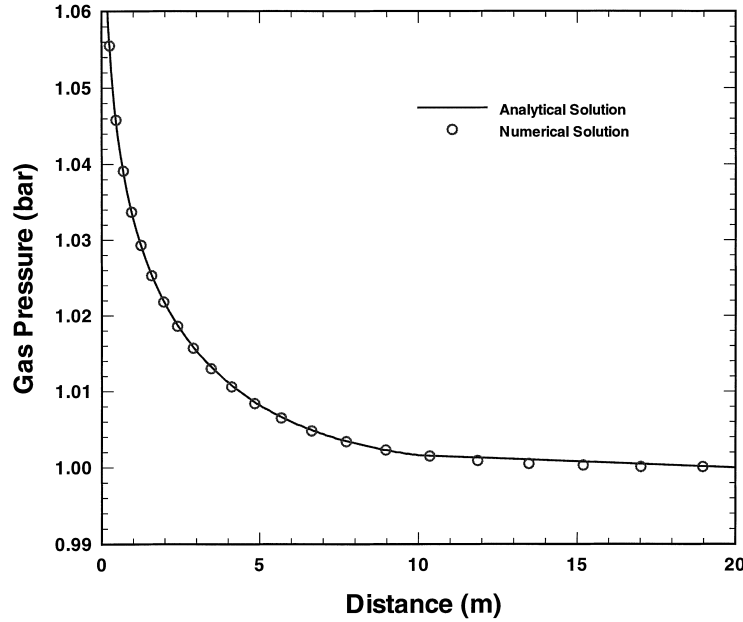


Figure 3. Comparison of the analytical and the numerical solutions for transient gas flow in a radial system.

average pressure, $1/\bar{P}$ (instead of $1/P$), (Klinkenberg, 1941; Reda, 1987; Persoff and Hulen, 1996). Flow condition is steady-state, linear flow through a linear core. In this method, an averaged gas permeability k_a is plotted against the reciprocal of the arithmetic mean of the inlet and outlet pressures,

$$k_a = k_\infty \left(1 + \frac{b}{(P_0 + P_L)/2} \right). \quad (5.1)$$

The averaged gas permeability is evaluated from pressure and flow data using a non-Klinkenberg gas flow solution of compressible gas (Scheidegger, 1974),

$$k_a = \frac{2\mu L q_L^v P_L}{P_0^2 - P_L^2}. \quad (5.2)$$

Here the superscript v indicates that volumetric, not mass, flux, is to be evaluated at the outlet pressure. By equivalence of the two averaged permeabilities of (5.1) and (5.2), the two Klinkenberg constants, k_∞ and b , can be determined from 2 or more data.

The use of $(P_0 + P_L)/2$ to represent P in Equation (1.1) to estimate gas permeability, combined with non-Klinkenberg solution (5.2) appears questionable. The gas pressure profiles, for the case of one-dimensional, steady-state flow (Figure 2), is not linear, and effective gas permeability will vary along the length. Using the exact solution (3.2), we have evaluated the traditional Klinkenberg analysis, based on Equations (5.1) and (5.2). Interestingly, we have found that the traditional method

is very accurate in deriving the two constants of Klinkenberg effects as long as the ratio of, b/P , is not extremely large.

Here, we derive an alternative approach for determining both k_∞ and b from laboratory tests, based on the exact steady-state flow solutions (3.2). By evaluating Equation (3.2) at $x = 0$ one obtains, after some algebraic manipulation,

$$\frac{q_m \mu L}{\beta(P_0 - P_L)} = bk_\infty + k_\infty \left(\frac{P_0 + P_L}{2} \right). \quad (5.3)$$

To evaluate k_∞ and b experimentally, a series of measurements are made in which the outlet pressure P_L is held constant while P_0 is varied and q_m is measured. Then $Y = q_m \mu L / \beta(P_0 - P_L)$ is plotted against $X = (P_0 + P_L)/2$. Then k_∞ and b are evaluated from the slope and intercept of the plot of Y against X .

5.3. WELL-TEST DETERMINATION OF PERMEABILITY AND KLINKENBERG COEFFICIENT

The transient gas flow solutions of (3.12) or (3.14) can be used to design well tests to determine both the gas permeability, k_∞ , and the Klinkenberg coefficient, b . Here, we give an example to demonstrate how to use the analytical solutions from a single well test of a constant mass rate pumping or injection with the line source solution, (3.12). The pumping or injection testing procedure is (1) measure initial reservoir gas pressure, P_i ; (2) impose a constant mass pumping or injection rate at the well; and (3) measure several (at least two) wellbore pressures at different times (avoiding the early time after-flow or wellbore storage effects). The Klinkenberg coefficient, b , can be directly calculated as,

$$b = -\frac{P_n + P_i}{2} - \frac{\mu q_m}{4\pi h k_\infty \beta (P_n - P_i)} \text{Ei} \left(-\frac{r_w^2}{4\alpha t_n} \right) \quad (5.4)$$

where the permeability, k_∞ , is determined from the following nonlinear algebraic equation,

$$P_j - P_n + \frac{\mu q_m}{2\pi h k_\infty \beta} \left\{ \frac{\text{Ei}(-r_w^2/4\alpha t_j)}{(P_j - P_i)} - \frac{\text{Ei}(-r_w^2/4\alpha t_n)}{(P_n - P_i)} \right\} = 0. \quad (5.5)$$

In Equations (5.4) and (5.5), P_n and P_j are the wellbore pressures measured at two different times, $t = t_n$ and $t = t_j$, respectively.

This method can be demonstrated to analyze the simulated well test of the example problem in Section 5.1 (Figure 3) to determine the Klinkenberg coefficient, b , and gas permeability, k_∞ . From the numerical simulation, the well pressure $P_w = 1.05130 \times 10^5$ Pa at $t = 8.64 \times 10^4$ s, and $P_w = 1.06976 \times 10^5$ Pa at $t = 8.64 \times 10^5$ s. Substituting these pressure and time data into Equation (5.5), together with the parameters in Section 5.1, leaves k_∞ as the only unknown. The resulting nonlinear equation can be easily solved using a bi-section method, which gives $k_\infty = 9.98 \times 10^{-16}$ m². Substituting this permeability value into (5.4) will give the Klinkenberg coefficient

$b = 4.77 \times 10^5$ Pa. The actual values are $k_\infty = 1.0 \times 10^{-15}$ m² and $b = 4.75 \times 10^5$, and this indicates the proposed well test method is very accurate in determining these two Klinkenberg parameters.

5.4. LABORATORY TEST ANALYSIS

5.4.1. *Materials and Methods*

Steady-state gas flow experiments were conducted to test the model and to evaluate k_∞ and b . Two rock core samples were obtained from well NEGU-17, in The Geysers geothermal field. Three cylindrical plugs, 15 mm in diameter were taken from the samples using a diamond core bit, and the cylinder ends were machined flat and parallel with lengths ranging from 9 to 11 mm.

The plugs were mounted into 2 in long stainless steel tubing using Castall E-205 epoxy resin. They were then dried at 60°C for 5 days to remove all moisture. All three sample tubes were connected to a gas inlet manifold where nitrogen gas was applied at controlled pressures ranging from 120 to 380 kPa.

Gas exiting from the sample flowed through a 1 m long horizontally mounted 3.175 mm o.d., 0.559 mm wall clear nylon tubing. To measure the gas flow rate, a slug of dyed water was injected into the tubing before it was connected to the sample tube, and the displacement of the slug was used to measure the gas flow rate. By monitoring the position of the slugs in the exit tubes, we were assured that steady state had been reached before measuring the flow rate.

Leaks that would normally be insignificant may be significant when measuring very low gas flows. An advantage of this experimental system is that any gas leak upstream of the sample would not cause any error, as long as the pressure is accurately measured. The gas flow that is to be monitored is at ambient pressure, so there is no driving force for it to leak and escape from the measurement tube. To test whether the technique of sealing the plugs into the stainless steel sample tube prevented gas from leaking past the sample, a dummy plug of aluminum was sealed into a stainless steel tube the same way and flow tested; no flow was observed.

5.4.2. *Results and Data Analysis*

The flow rate and pressure data are summarized in Table II. These data will be interpreted according to the traditional Klinkenberg method of (5.1) and (5.2) and to the new model (5.3), referred as to exact Klinkenberg analysis in this paper. In both cases, the data of Table II are used to calculate derived quantities that are plotted as straight lines.

In the exact Klinkenberg analysis, the calculated quantities $X = (P_0 + P_L)/2$ and $Y = q_m \mu L / \beta (P_0 - P_L)$ are summarized in Table II and are plotted in Figure 4 for the three samples. In the traditional Klinkenberg analysis, Figure 5 plots the data calculated in Table II, and Table III presents the calculated values of k_∞ and b derived from the linear plots, as well as the correlation coefficients. The values obtained by

Table II. Steady-state gas flow measurements on plugs of The Geysers Greywacke from well NEGU-17

Sample dimensions	Raw data			Quantities calculated for traditional analysis		Quantities calculated for exact analysis	
	Inlet pressure	Outlet pressure	Volume flow rate at exit pressure	Inverse pressure	k_a	$X = \frac{P+P_L}{2}$	$\frac{q_m \mu L}{\beta(P_0 - P_L)}$
	(Pa)	(Pa)	(m ³ /s)	(Pa ⁻¹)	(m ²)	(Pa)	(N)
Sample 36 $A = 1.79E - 04 \text{ m}^2$ $L = 9.07E - 03 \text{ m}$	2.18E + 05	9.88E + 04	4.48E - 11	6.31E - 06	2.07E - 19	1.59E + 05	3.28E - 14
	2.64E + 05	9.88E + 04	6.24E - 11	5.51E - 06	1.82E - 19	1.82E + 05	3.30E - 14
	3.05E + 05	9.88E + 04	7.88E - 11	4.95E - 06	1.66E - 19	2.02E + 05	3.34E - 14
	3.40E + 05	9.87E + 04	9.26E - 11	4.56E - 06	1.53E - 19	2.19E + 05	3.35E - 14
	3.80E + 05	9.93E + 04	1.09E - 10	4.17E - 06	1.42E - 19	2.40E + 05	3.40E - 14
	1.42E + 05	9.95E + 04	1.54E - 11	8.29E - 06	2.65E - 19	1.21E + 05	3.20E - 14
Sample 9a $A = 1.83E - 04 \text{ m}^2$ $L = 1.04E - 02 \text{ m}$	2.18E + 05	9.88E + 04	5.41E - 11	6.31E - 06	2.79E - 19	1.59E + 05	4.43E - 14
	2.64E + 05	9.88E + 04	7.58E - 11	5.51E - 06	2.47E - 19	1.82E + 05	4.48E - 14
	3.05E + 05	9.88E + 04	9.53E - 11	4.95E - 06	2.24E - 19	2.02E + 05	4.53E - 14
	3.40E + 05	9.87E + 04	1.11E - 10	4.56E - 06	2.05E - 19	2.19E + 05	4.50E - 14
	3.80E + 05	9.93E + 04	1.31E - 10	4.17E - 06	1.91E - 19	2.40E + 05	4.58E - 14
	1.42E + 05	9.95E + 04	1.81E - 11	8.29E - 06	3.49E - 19	1.21E + 05	4.22E - 14
Sample 9b $A = 1.83E - 04 \text{ m}^2$ $L = 1.04E - 02 \text{ m}$	2.18E + 05	9.88E + 04	6.16E - 11	6.31E - 06	3.18E - 19	1.59E + 05	5.04E - 14
	2.64E + 05	9.88E + 04	8.66E - 11	5.51E - 06	2.82E - 19	1.82E + 05	5.12E - 14
	3.05E + 05	9.88E + 04	1.10E - 10	4.95E - 06	2.58E - 19	2.02E + 05	5.20E - 14
	3.40E + 05	9.87E + 04	1.25E - 10	4.56E - 06	2.31E - 19	2.19E + 05	5.07E - 14
	3.80E + 05	9.93E + 04	1.52E - 10	4.17E - 06	2.22E - 19	2.40E + 05	5.32E - 14
	1.42E + 05	9.95E + 04	2.07E - 11	8.29E - 06	3.99E - 19	1.21E + 05	4.82E - 14
	1.20E + 05	9.91E + 04	1.01E - 11	9.12E - 06	4.30E - 19	1.10E + 05	4.71E - 14

the two methods are close, although the traditional plot appears to have a better correlation.

When two constants are to be determined from more than two measurements (i.e. data are redundant), fitting the data to a linear equation using least squares generally provides the best estimates of the constants. But if more than one linearization is possible, the same data set will yield different results depending upon the linearization chosen (see, for example, Persoff and Thomas, 1988). It is tempting to prefer the linearization that yields the values of r^2 closer to unity. However, note that in the traditional method, the value of k_a (which is plotted as the dependent variable) calculated from (5.2) includes a factor of $1/(P_0 + P_L)$ which is just double the independent variable. This artificially inflates the value of r^2 .

Table III. Analysis results of the laboratory tests

Sample	Traditional			Exact		
	k_{∞} (m ²)	b (Pa)	r^2	k_{∞} (m ²)	b (Pa)	r^2
36	1.66E-20	1.81E+06	1.00E+00	1.61E-20	1.88E+06	9.88E-01
9a	3.15E-20	1.23E+06	9.99E-01	2.75E-20	1.43E+06	9.33E-01
9b	4.38E-20	9.75E+05	9.99E-01	4.02E-20	1.08E+06	9.28E-01

where r^2 is correlation coefficient.

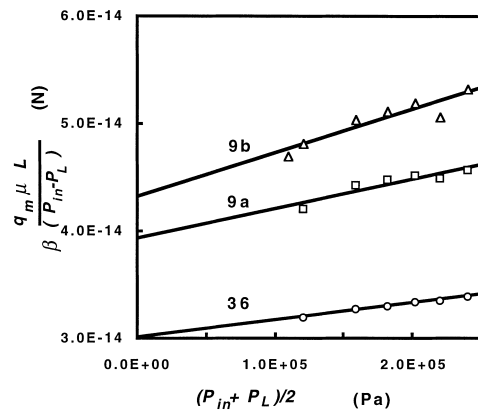


Figure 4. Exact Klinkenberg analysis plot for the three test samples.

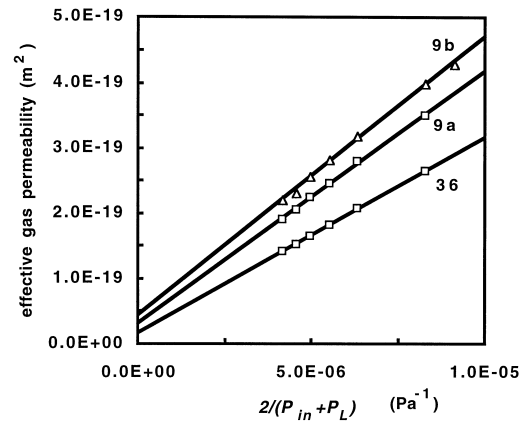


Figure 5. Traditional Klinkenberg analysis plot for the three test samples.

6. Concluding Remarks

A general gas flow governing equation including the Klinkenberg effect has been derived by introducing a new pressure variable. Based on this new form of gas flow governing equation, a set of new analytical solutions has been developed for analyzing steady-state and transient gas flow through porous media with Klinkenberg effects. As an extension of this work, additional analytical solutions for one-, two- and three-dimensional gas flow with the Klinkenberg effect can be readily derived. These analytical solutions will find their applications in analyzing gas flow and determining soil flow properties in the unsaturated zone or in laboratory tests where the Klinkenberg effects cannot be ignored.

To determine the condition under which the linearized gas flow equation may be applicable, a numerical method is used to examine the predictions from the approximate analytical solutions for transient gas flow. It has been found that the conventional linearization procedure of deriving gas flow equations, using an initial gas pressure for the diffusivity term, will result in acceptable solutions when the overall pressure variations in the system are small. However, the linearization assumption may introduce considerable errors when pressure changes are significantly different from the ambient condition. In this case, we propose a new evaluation procedure for the diffusivity term using a history-dependent averaged pressure with analytical solutions, which will still give accurate solutions even under high pressure disturbed conditions of the system.

In order to demonstrate their applications, the new analytical solutions have been used to verify the numerical solutions of gas flow which include the Klinkenberg effect. Several new laboratory and field testing techniques are derived, based on the analytical solutions for determining the Klinkenberg parameters of porous medium gas flow. These new laboratory and field test analysis methods are very easy to implement and more accurate to use. One of the proposed laboratory methods has been applied to laboratory testing results in determining absolute permeability and Klinkenberg constants and to examination of the traditional Klinkenberg analysis. The transient test analysis method is illustrated using a simulated well test result.

Appendix: Derivation of the Gas Flow Equation

Under isothermal conditions, gas flow in porous media is governed by a mass balance equation,

$$\nabla \cdot (\rho \mathbf{v}) = -\phi \frac{\partial(\rho)}{\partial t}, \quad (\text{A.1})$$

where ρ is gas density; ϕ is formation porosity, assumed to be constant; \mathbf{v} is the Darcy's velocity of the gas phase, defined as

$$\mathbf{v} = -\frac{k_g}{\mu} (\nabla P - \rho \mathbf{g}), \quad (\text{A.2})$$

where μ is gas-phase viscosity; \mathbf{g} is gravity vector; and k_g is effective gas-phase permeability, described by Equation (1.1), including the Klinkenberg effects.

The ideal gas law is here used to describe the relation between gas density and pressure as,

$$\rho = \beta P, \quad (\text{A.3})$$

where β is a compressibility factor, defined as

$$\beta = M_g / RT \quad (\text{A.4})$$

with M_g being the molecular weight of the gas; R the universal gas constant; and T constant temperature.

When gravity effects are ignored, combining Equations (A.1)–(A.3), and (1.1) will give

$$\nabla \cdot \left(\frac{k_\infty \beta}{\mu} (P + b)(\nabla P) \right) = \phi \beta \frac{\partial P}{\partial t}. \quad (\text{A.5})$$

In terms of the new variable, $P_b = P + b$, Equation (A.5) may be written as the form of (2.1).

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