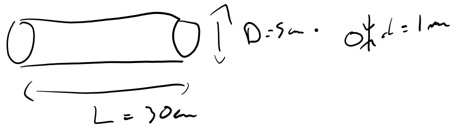


$$V_s = \frac{\pi D^2 L}{4} = 589\ \text{cm}^3 \qquad \varepsilon = \frac{V_{air}}{V_s} = \frac{V_s - V_{all\,grain}}{V_s}$$

$$V_{all\,grain} = (1-\varepsilon)V_s = 377\ \text{cm}^3$$

number of grains

$$v_{1g} = \frac{\pi d^3}{6} = 5{,}2\times10^{-4}\ \text{cm}^3 \qquad n = \frac{V_{all\,grain}}{v_{1g}} = 720\times10^3$$

$r \ll L \rightarrow$ separation of scales.

$$\delta = \frac{r}{L} \ll 1$$

(V) physical quantities check the scale separation

We introduce 2 variables: $\frac{\vec{X}}{r}$ and $\frac{\vec{X}}{L}$

For convenience: $\vec{x}$ and $\vec{y} = \delta\vec{x}$ $\quad (\vec{y} \ll \vec{x})$

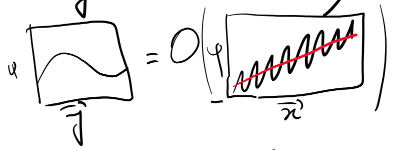
We can write $\varphi(\vec{X}) = \varphi(\vec{x}, \vec{y})$

We denote $\langle\varphi\rangle$ spatial average of φ over the REV

$$\varphi = O(\langle\varphi\rangle) \iff \delta \ll \frac{\varphi}{\langle\varphi\rangle} \ll \frac{1}{\delta}$$

consider the gradients

$$\frac{\partial\varphi}{\partial\vec{y}} = O\left(\frac{\partial\langle\varphi\rangle}{\partial\vec{x}}\right) = O\left(\frac{\partial\varphi}{\partial\vec{x}}\right)$$



$\Rightarrow$ We can write an asymptotic development:

$$\varphi(\vec{x},\vec{y}) = \varphi^{(0)}(\vec{x},\vec{y}) + \delta\varphi^{(1)}(\vec{x},\vec{y}) + \delta^2\varphi^{(2)}(\vec{x},\vec{y}) + \dots$$

$\varphi^{(i)}(\vec{x},\vec{y})$ are $\vec{y}$-stationary

if $\delta \to 0$, $\varphi(\vec{x},\vec{y}) = \langle\varphi\rangle$.

Volume averaging

$$\langle\varphi_\alpha\rangle^\alpha = \frac{V}{V_\alpha}\langle\varphi_\alpha\rangle, \qquad \frac{V}{V_\alpha} = \frac{1}{\varepsilon_\alpha}$$

$$\Rightarrow \underbrace{\langle\varphi_\alpha\rangle^\alpha}_{\text{intrinsic}} = \varepsilon_\alpha^{-1}\underbrace{\langle\varphi_\alpha\rangle}_{\text{superficial}}$$

Interfacial integrals

$\varphi = \langle\varphi\rangle^\alpha + \tilde{\varphi}$ $\qquad \langle\cdot\rangle$ = average over the REV

$$\Rightarrow \frac{1}{V}\int_{S_{\alpha\sigma}}\varphi\,\vec{n}_{\alpha\sigma}\,dS = \underbrace{\frac{1}{V}\int_{S_{\alpha\sigma}}\langle\varphi\rangle^\alpha\,\vec{n}_{\alpha\sigma}\,dS}_{I} + \frac{1}{V}\int_{S_{\alpha\sigma}}\tilde{\varphi}\,\vec{n}_{\alpha\sigma}\,dS$$

$$I = \frac{1}{V}\int_{S_{\alpha\sigma}}\langle\varphi\rangle^\alpha\,\vec{n}_{\alpha\sigma}\,dS = \frac{\langle\varphi\rangle^\alpha}{V}\underbrace{\int_{S_{\alpha\sigma}}\vec{n}_{\alpha\sigma}\,dS}_{?}$$

we consider φ uniform

$$\Rightarrow \langle\vec{\text{grad}}\,\varphi\rangle = \vec{0} = \vec{\text{grad}}\langle\varphi\rangle + \frac{\varphi}{V}\int_{S_{\alpha\sigma}}\vec{n}_{\alpha\sigma}\,dS$$

Moreover, $\langle\varphi\rangle = \varepsilon_\alpha\langle\varphi\rangle^\alpha$

and $\vec{\text{grad}}(fg) = f\,\vec{\text{grad}}\,g + g\,\vec{\text{grad}}\,f$.

$$\frac{\varphi}{V}\int_{S_{\alpha\sigma}}\vec{n}_{\alpha\sigma}\,dS = -\left(\varepsilon_\alpha\,\vec{\text{grad}}\langle\varphi\rangle^\alpha + \langle\varphi\rangle^\alpha\,\vec{\text{grad}}\,\varepsilon_\alpha\right)$$

and $\langle\varphi\rangle^\alpha = \frac{1}{V_\alpha}\int_{V_\alpha}\varphi\,dV$

$$\Rightarrow -\frac{1}{V}\int_{S_{\alpha\sigma}}\vec{n}_{\alpha\sigma}\,dS = \frac{\vec{\text{grad}}\,\varepsilon_\alpha}{\varphi V_\alpha}\int_{V_\alpha}\varphi\,dV + \frac{\varepsilon_\alpha}{\varphi}\,\vec{\text{grad}}\cancel{\left(\frac{1}{V_\alpha}\int_{V_\alpha}\varphi\,dV\right)}$$

Remind: φ is uniform $\Rightarrow \int_{V_\alpha}\varphi\,dV = \varphi V_\alpha$

$$\Rightarrow \frac{1}{V}\int_{S_{\alpha\sigma}}\vec{n}_{\alpha\sigma}\,dS = -\vec{\text{grad}}\,\varepsilon_\alpha$$

$$\Rightarrow I = \frac{1}{V}\int_{S_{\alpha\sigma}}\langle\varphi\rangle^\alpha\,\vec{n}_{\alpha\sigma}\,dS = -\langle\varphi\rangle^\alpha\,\vec{\text{grad}}\,\varepsilon_\alpha.$$

$$\Rightarrow \frac{1}{V}\int_{S_{\alpha\sigma}}\varphi\,\vec{n}_{\alpha\sigma}\,dS = -\langle\varphi\rangle^\alpha\,\vec{\text{grad}}\,\varepsilon_\alpha + \frac{1}{V}\int_{S_{\alpha\sigma}}\tilde{\varphi}\,\vec{n}_{\alpha\sigma}\,dS.$$