MFEE department
3ème année - Master FEIP
Toulouse INP – ENSEEIHT

Transfers in Porous Media
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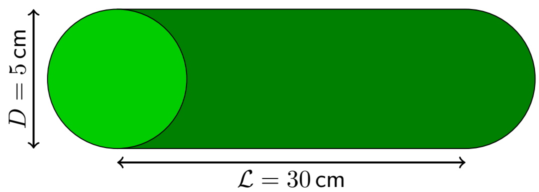
Upscaling to porous media

Exercises

1 Sand grains in a core sample



1. Sand grains are 1 mm in diameter, mono-disperse. Assuming the dimensions from the above picture, how many sand grains are in this core sample? We assume that the porosity corresponds to a random packing ($\epsilon = 0.36$).

2 Permeability of a composite porous media

We want to assess the permeability of a composite porous media. The permeability is the ability for a porous media to let a fluid pass through when a pressure difference ΔP iss applied along the porous media. We consider a flow rate Q passing through a fully saturated porous media. For a homogeneous porous media of length L and cross-section S, we define the permeability k as

$$Q = \frac{k}{\eta}\frac{S}{L}\Delta P. \tag{1}$$

Figure 1 shows a porous media composed of N alternative layers whose thickness and permeability are respectively (h_1, k_1) and (h_2, k_2). The fluid has a viscosity η. Two configurations (a) and (b) (see figure 1) are considered. We assume that $L \gg h_1, h_2$.

1. For the first configuration (a) the pressure drop is parallel to one of the cube sides and to the layers. Give the expression of the flow rate through the faces perpendicular to this side.

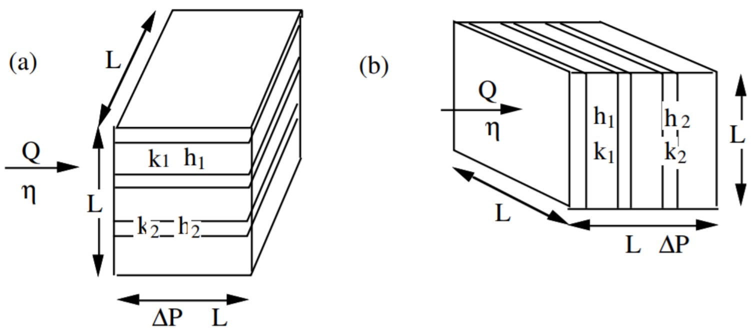


Figure 1: Composite porous media in two configurations relatively to the mass flux. Adapted from F. Moisy.

2. Deduce the expression of an equivalent permeability $k_{||}$.

3. For the second configuration (b) perpendicularly to the layers. Give the expression of the flow rate through the faces parallel to the layers.

4. Deduce the expression of an equivalent permeability $k_{\perp}$.

5. For both cases, what happens if k_1 or k_2 becomes zero or infinity?

3 Volume-averaging of the hydrostatics equation

To start using the volume-averaging method, we will apply it to a simple local equation: the hydrostatics equation.

$$-\overrightarrow{\text{grad}} P + \rho_\alpha \vec{g} = \vec{0}, \tag{2}$$

where P is the pressure and α index or exponent is related to fluid α phase. The boundary condition is $P = P_0$ for $\vec{r_\alpha} = \vec{0}$ where $\vec{r_\alpha}$ locates points in the α-phase as illustrated in figure 2.

1. We remind the definition of the gradient: $dP = \overrightarrow{\text{grad}} P \cdot \overrightarrow{dr_\alpha}$. Integrate the local hydrostatics equation to get the local pressure equation.

2. Remind the definition of the intrinsic average.

3. Using the intrinsic average we get:

$$\langle P \rangle^\alpha = \rho \langle \vec{r_\alpha} \rangle^\alpha \cdot \vec{g} + P_0. \tag{3}$$

Develop this equation with the geometrical decomposition of $\vec{r_\alpha}$.

4. Apply the gradient operator to the previously obtained equation. By noticing that the gradient here represents the derivative with respect to $\vec{x}$ show that:

$$\overrightarrow{\text{grad}} \langle P \rangle^\alpha = \rho \vec{g} + \rho \overrightarrow{\text{grad}} \langle \vec{y_\alpha} \rangle^\alpha \cdot \vec{g}. \tag{4}$$

5. We note r_0 the typical scale of the REV, $\mathcal{L}$ the macroscale and ℓ_α the pore typical size. Which hypothesis is necessary to carry on volume averaging ?

6. Finally, use this hypothesis to make an approximation and simplify the previous equation.

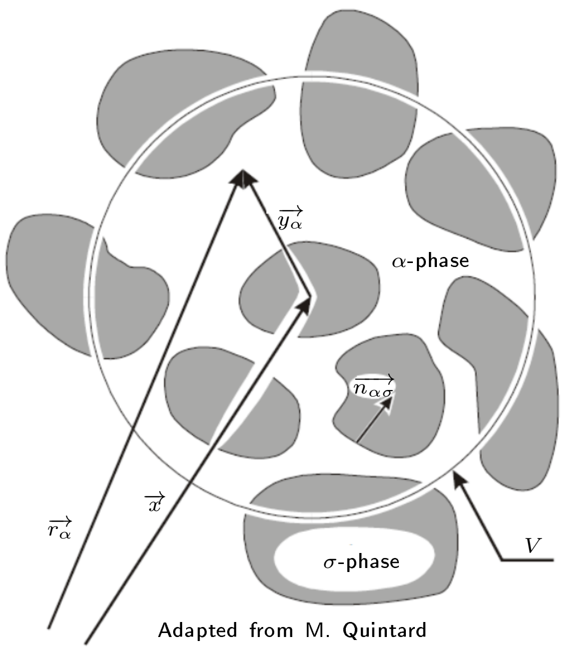


Figure 2: Sketch of the REV.