Upscaling to porous media

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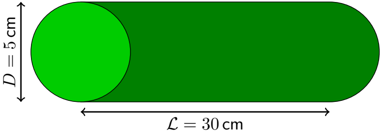
Toulouse INP N7
IMFT Institut de Mécanique des Fluides de Toulouse

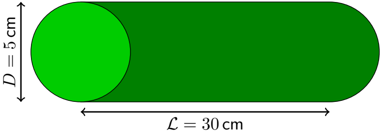
An order of magnitude...

‘

> Core sample: sample used to characterize properties of a porous media (ex: soil)



- Core sample: sample used to characterize properties of a porous media (ex: soil)



Take your smartphone

An order of magnitude...

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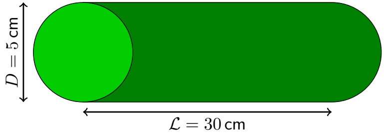
#QDLE#S#ABCD##



In your opinion, an order of magnitude of the number of sand grains (1 mm in diameter, $\epsilon = 0.36$) in the above sample?

A. Several hundreds

B. Several thousands

C. Several tens of thousands

D. Several hundred of thousands

An order of magnitude...

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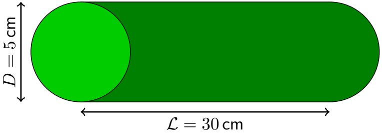

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Exercise: the number of sand grains in this sample

An order of magnitude...

Take your smartphone

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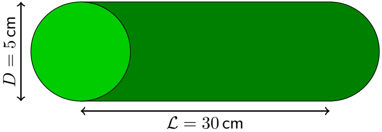
#QDLE#Q#ABCD*##



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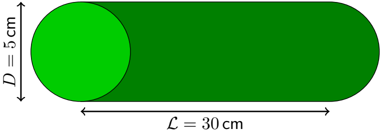

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Exercise: the number of sand grains in this sample

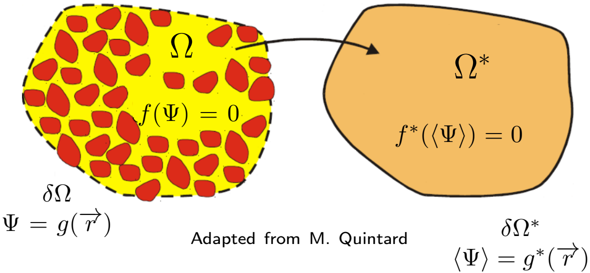
Problem: we cannot apply physics law (Navier-Stokes, diffusion, ...) on the whole porous media.

> Objective 1: rapid and precise computation on a macroscopic porous media

⇒ Reduction of the number of degrees of freedom

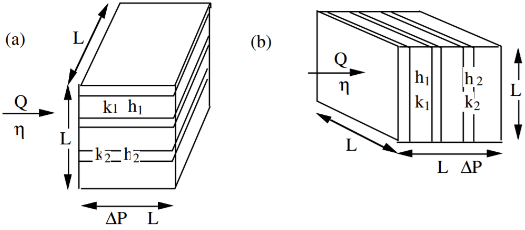
> Objective 2: get macroscopic equations and limit conditions, effective properties

⇒ From heterogeneous to homogeneous scale



Adapted from M. Quintard

Exercise: compute the permeability of the material below in both configurations



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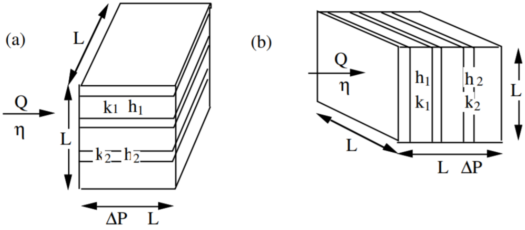
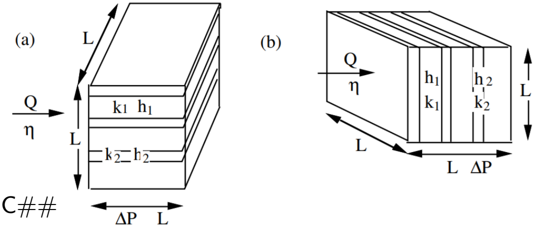


A first example of simple heterogeneous media

Exercise: compute the permeability of the material below in both configurations



#QDLE#Q#AB*C##

What is the expression of $k_{\parallel}$?

A. $k_{\parallel} = \dfrac{h_1 k_2 + h_2 k_1}{h_1 + h_2}$

B. $k_{\parallel} = \dfrac{h_1 k_1 + h_2 k_2}{h_1 + h_2}$

C. $k_{\parallel} = (h_1 k_1 + h_2 k_2)(h_1 + h_2)$

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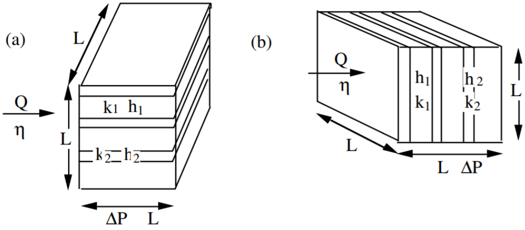

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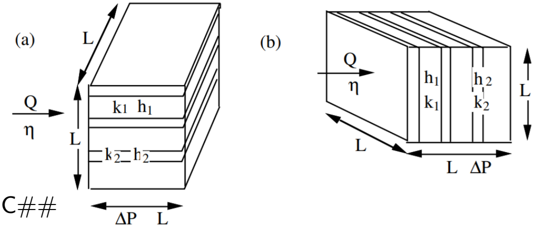

#QDLE#Q#A*BC##

What is the expression of $k_\perp$?

A. $k_\perp = \dfrac{h_1 + h_2}{h_1/k_1 + h_2/k_2}$

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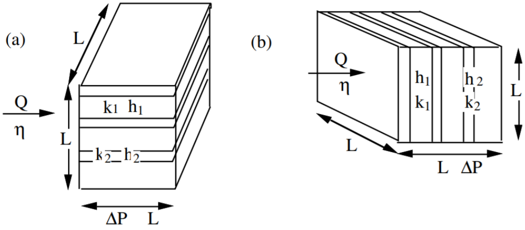

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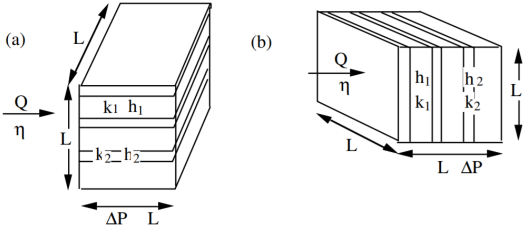
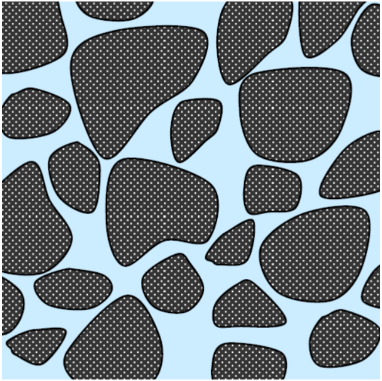
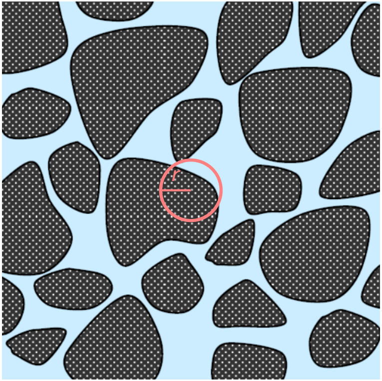


The equivalent permeability is not always a simple average of the permeabilities of each phase !




$\langle \Psi \rangle$
r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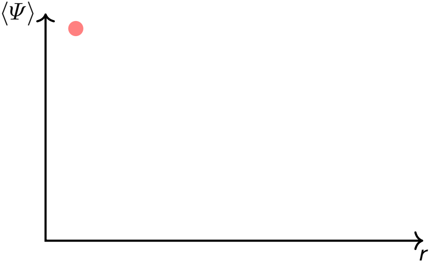
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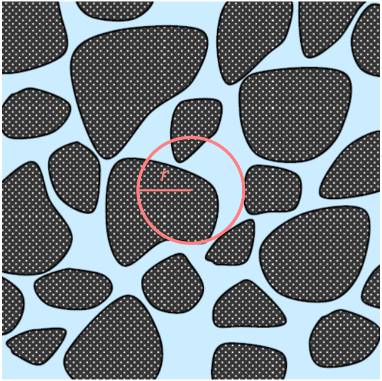
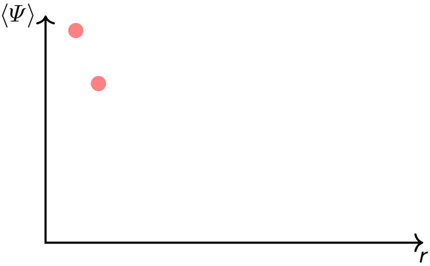


r



$\langle \Psi \rangle$
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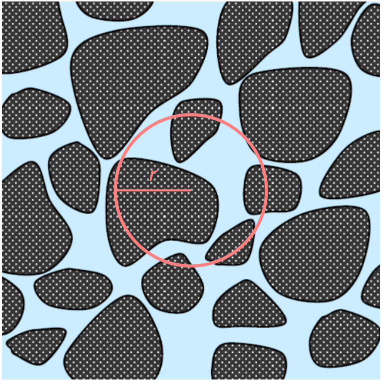
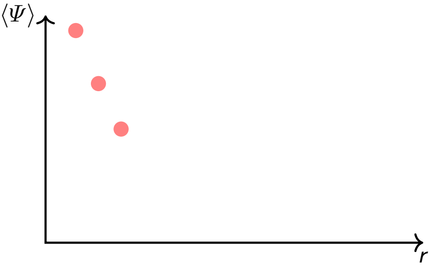
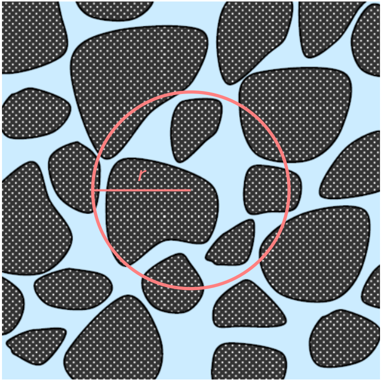
The *Representative Elementary Volume (REV)*



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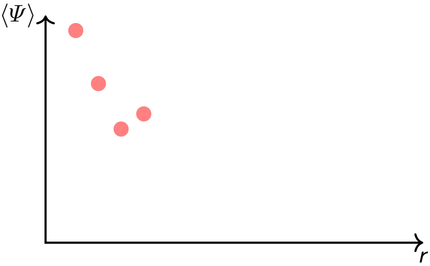
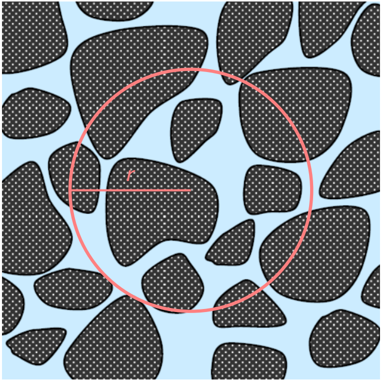

$\langle\Psi\rangle$
r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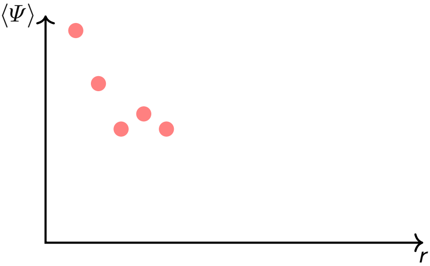
$\langle \Psi \rangle$
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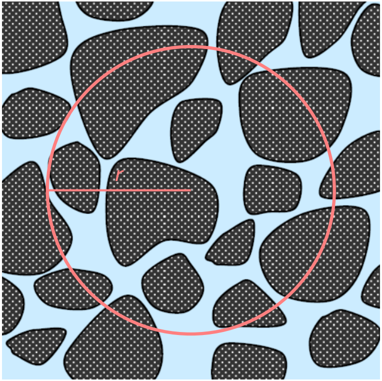

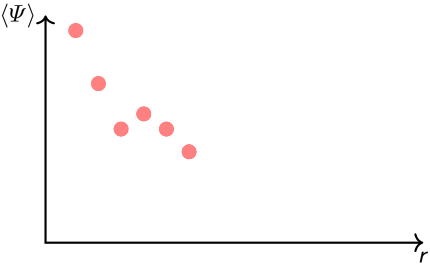

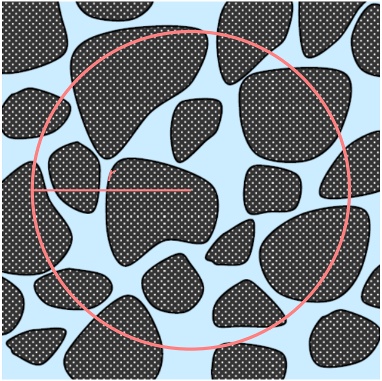
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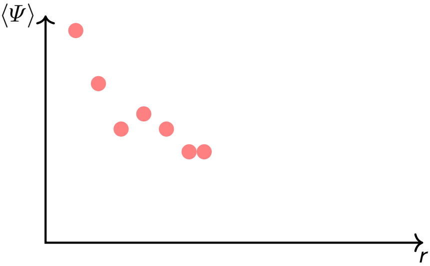
$\langle \Psi \rangle$
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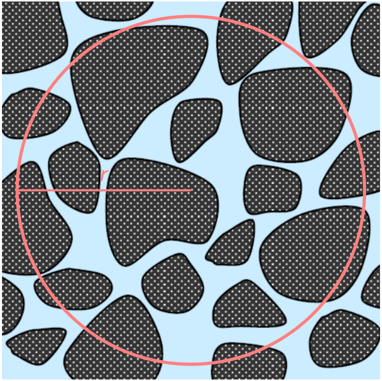
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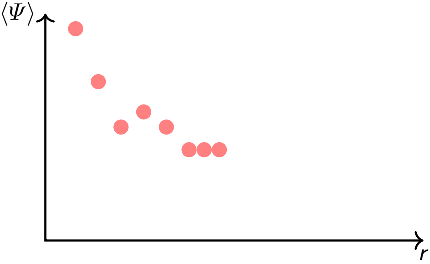
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r

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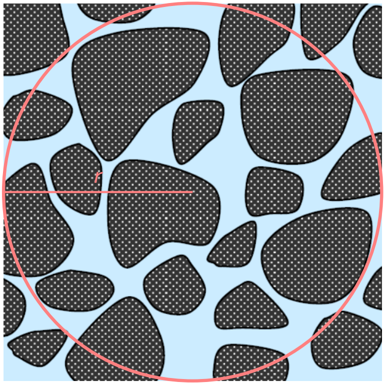

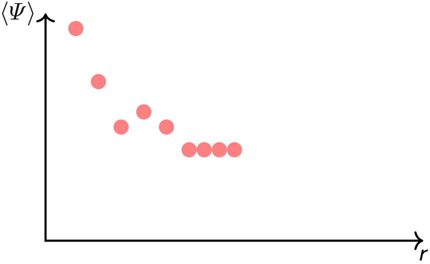

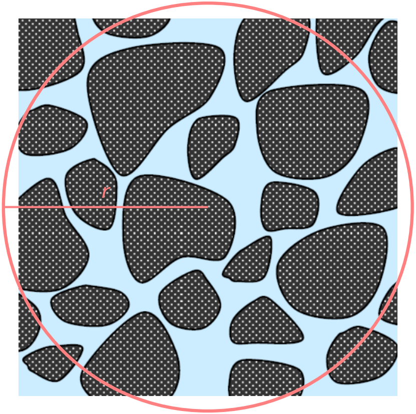

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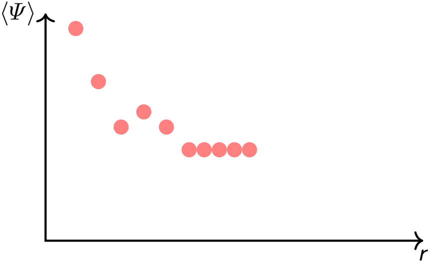

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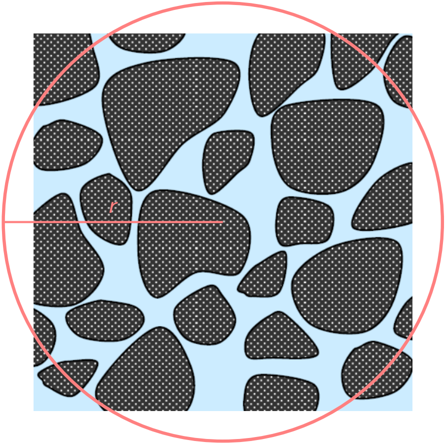

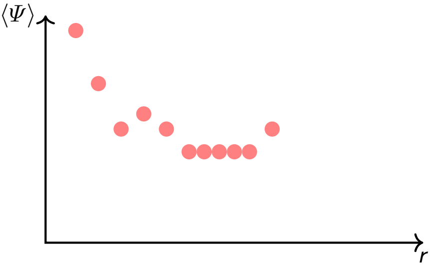

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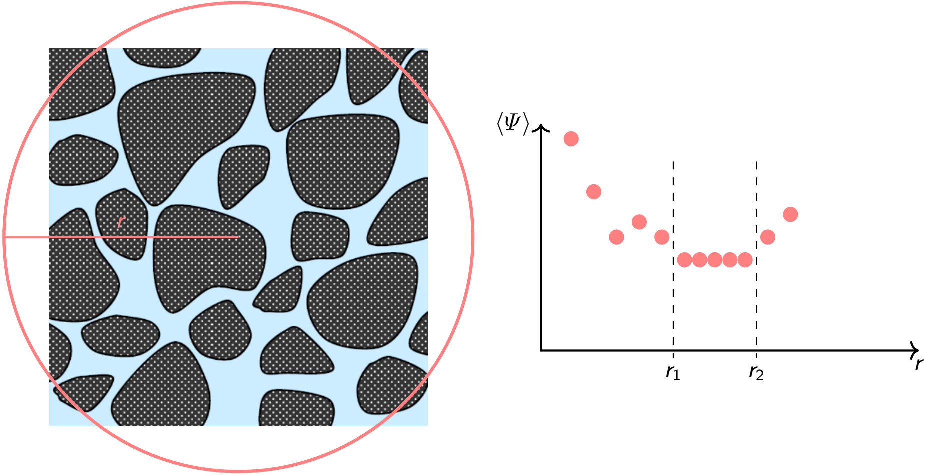

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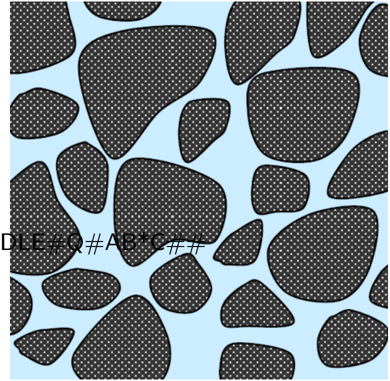

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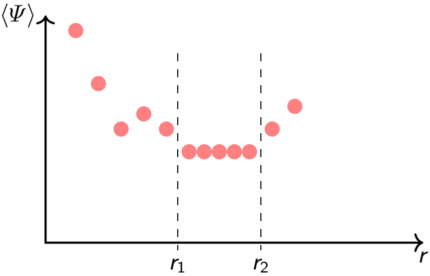
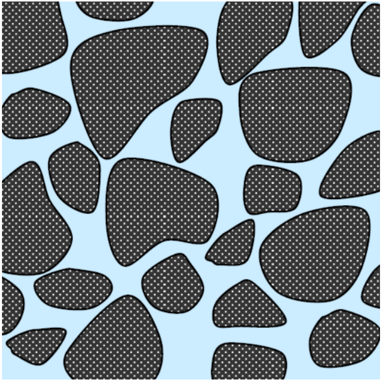


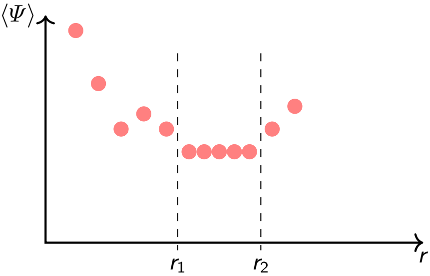
What is the representative elementary volume?

A. $r < r_1$

B. $r_1 < r < r_2$

C. $r_2 < r$





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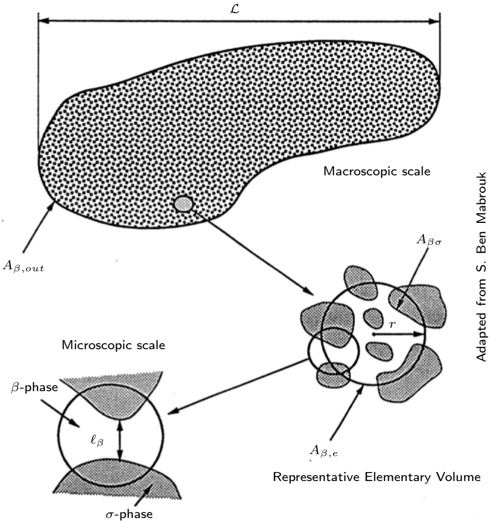
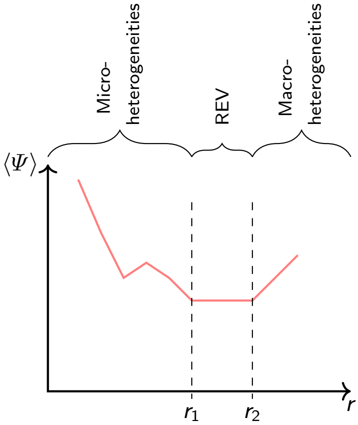
The REV is a volume range where porous media properties are about constant while moving the volume.

The *Representative Elementary Volume (REV)*



Adapted from S. Ben Mabrouk



> For porosity: $r \sim 4 - 5\ell_\beta$

Objective: Get macroscopic transfer laws from microscopic ones

> Method proposed by Whitaker and Quintard (90's)

Hypothesis:

> Existence of a structural REV

Homogenization of the local laws: volume averaging

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- Local equilibrium at REV scale
  - Variable state quasi uniform in the REV (pressure, temperature, ...)
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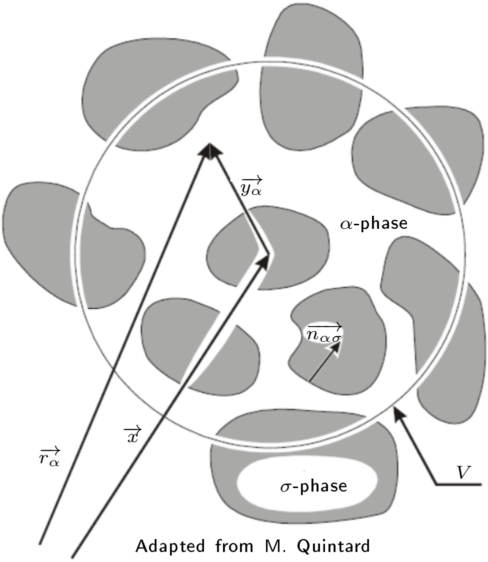
- Local equilibrium at REV scale
  - Variable state quasi uniform in the REV (pressure, temperature, ...)
  - No constraint on flux variables (velocity, heat flux, ...)
  - Fluid phases distribution driven by quasi-static capillary effects
- Scale separation
  - Laws at pore scale: average of molecular dynamics
  - REV: average on a small piece of porous media
  - Macroscopic: average on moving EVR
  - These three scales must be separated, including physical quantities



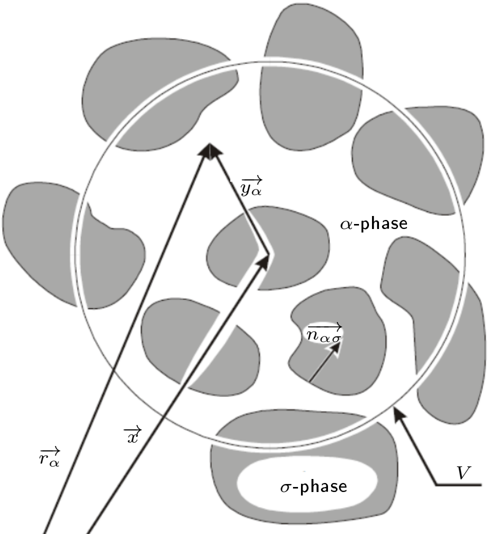
Adapted from M. Quintard

Averaging on the REV

Superficial average

$$\langle \Psi_\alpha \rangle = \frac{1}{V} \int_{V_\alpha} \Psi_\alpha(\overrightarrow{x} + \overrightarrow{y_\alpha}) dV$$

Homogenization of the local laws: volume averaging



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Averaging on the REV

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$$\langle \Psi_\alpha \rangle = \frac{1}{V} \int_{V_\alpha} \Psi_\alpha(\overrightarrow{x} + \overrightarrow{y_\alpha}) dV$$

Intrinsic average

$$\langle \Psi_\alpha \rangle^\alpha = \frac{1}{V_\alpha} \int_{V_\alpha} \Psi_\alpha(\overrightarrow{x} + \overrightarrow{y_\alpha}) dV$$

- Relation: $\langle \Psi_\alpha \rangle = \epsilon_\alpha \langle \Psi_\alpha \rangle^\alpha$
- ϵ_α: α-phase volume fraction

Theorem 1 – Gradient spatial averaging

$$\langle \overrightarrow{\mathrm{grad}} \Psi \rangle = \overrightarrow{\mathrm{grad}} \langle \Psi \rangle + \frac{1}{V} \int_{S_{\alpha\sigma}} \Psi \overrightarrow{n_{\alpha\sigma}} dS$$

Homogenization of the local laws: volume averaging

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Theorem 2 – Divergence spatial averaging

$$\langle \mathrm{div} \overrightarrow{\Psi} \rangle = \mathrm{div} \langle \overrightarrow{\Psi} \rangle + \frac{1}{V} \int_{S_{\alpha\sigma}} \overrightarrow{\Psi} \cdot \overrightarrow{n_{\alpha\sigma}} dS$$

Theorem 1 – Gradient spatial averaging

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Theorem 2 – Divergence spatial averaging

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> $\frac{1}{V} \int_{S_{\alpha\sigma}} \Psi \overrightarrow{n_{\alpha\sigma}} dS$: interfacial integral (tortuosity, ...)

> $\Psi = \langle \Psi \rangle^{\alpha} + \tilde{\Psi}$ (average + fluctuations)

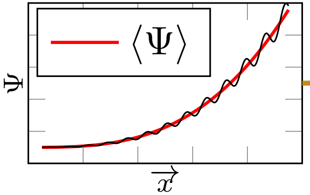
Under scale separation assumption, we have:

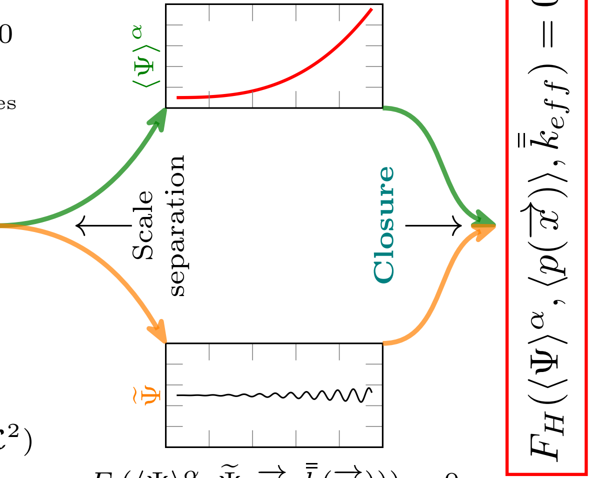
$$\frac{1}{V} \int_{S_{\alpha\sigma}} \Psi \overrightarrow{n_{\alpha\sigma}} dS = -\langle \Psi \rangle^{\alpha} \overrightarrow{\text{grad}} \epsilon_{\alpha} + \frac{1}{V} \int_{S_{\alpha\sigma}} \tilde{\Psi} \overrightarrow{n_{\alpha\sigma}} dS$$

$$F(\Psi, p(\vec{x}), \bar{\bar{k}}(\vec{x})) = 0$$

$$\begin{cases} p(\vec{x}): & \text{forcing} \\ \bar{\bar{k}}(\vec{x}): & \text{media properties} \end{cases}$$

$$F_{\langle \cdot \rangle}(\langle \Psi \rangle^{\alpha}, \widetilde{\Psi}, \vec{x}, \bar{\bar{k}}(\vec{x}))) = 0$$





$$F_{\tilde{\cdot}}(\langle \Psi \rangle^{\alpha}, \widetilde{\Psi}, \vec{x}, \bar{\bar{k}}(\vec{x}))) = 0$$

Closure

$$\widetilde{\Psi} = g(\langle \Psi \rangle^{\alpha}) + \mathcal{O}(r^2/\mathcal{L}^2)$$

$$\begin{cases} g: & \text{linear relation} \\ \Rightarrow \bar{\bar{k}}_{eff}: & \text{effective properties} \end{cases}$$

A simple volume-averaging example

Let's make the volume averaging of the hydrostatics local equation :

$$-\overrightarrow{\mathrm{grad}} P + \rho \overrightarrow{g} = \overrightarrow{0}$$

A simple volume-averaging example

Let's make the volume averaging of the hydrostatics local equation :

$$-\overrightarrow{\mathrm{grad}}P + \rho\,\overrightarrow{g} = \overrightarrow{0}$$

How can we write the intrinsic average of P as a function of $\overrightarrow{y_\alpha}$?

A. $\langle P\rangle^\alpha = \rho\,\overrightarrow{x}\cdot\overrightarrow{g} + \rho\langle\overrightarrow{y_\alpha}\rangle^\alpha\cdot\overrightarrow{g}$

#QDLE#Q#AB*C##

B. $\boldsymbol{\langle P\rangle^\alpha = \rho\,\overrightarrow{x}\cdot\overrightarrow{g} + \rho\langle\overrightarrow{y_\alpha}\rangle^\alpha\cdot\overrightarrow{g} + P_0}$

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A simple volume-averaging example

Let's make the volume averaging of the hydrostatics local equation :

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How can we write the intrinsic average of P as a function of $\overrightarrow{y_\alpha}$?

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Objective: Get macroscopic transfer laws from microscopic ones

- Method proposed by Sanchez-Palencia and Auriault (80's – 90's)

Hypothesis:

- Existence of a REV with typical dimension r
- Separation of scales: $r \ll \mathcal{L}$ ($\mathcal{L}$: macroscopic scale)
- Separation of scales **must be valid for physical values**

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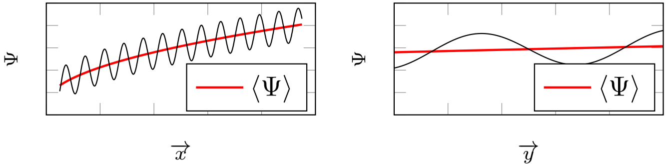

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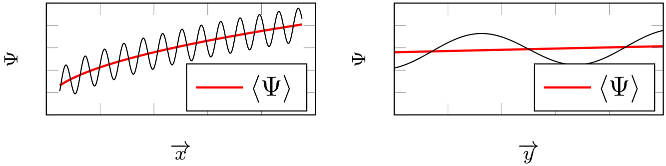

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Asymptotic development

$$\Psi(\vec{x},\vec{y}) = \Psi^{(0)}(\vec{x},\vec{y}) + \delta\Psi^{(1)}(\vec{x},\vec{y}) + \delta^2\Psi^{(2)}(\vec{x},\vec{y}) + \dots, \quad (0.1)$$

> $\Psi^{(i)}(\vec{x},\vec{y})$: $\vec{y}$-stationary