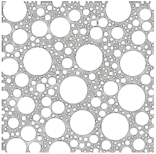
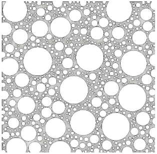


Lecture presentation: prerequisites and proceedings



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Self-assessment

Take your smartphone

Self-assessment – Hydrostatics

- $\vec{g}$: acceleration due to gravity
- ρ: density
- P: pressure

What is the fundamental equation of hydrostatics?

A. $\overrightarrow{\mathrm{grad}}\rho = P\vec{g}$

B. $\overrightarrow{\mathrm{grad}}P = -\rho\vec{g}$

C. $\overrightarrow{\mathrm{grad}}P = \rho\vec{g}$

#QDLE#Q#ABC*##

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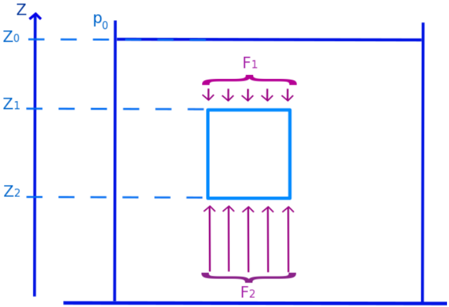
> The pressure gradient has the same orientation as the gravity
> Points of fluid at the same altitude: same hydrostatic pressure



- γ: surface tension
- r: drop radius
- P_i: pressure inside
- P_o: pressure outside

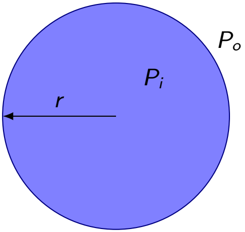
What is the pressure difference between the two sides of a drop interface ?

A. $P_o - P_i = \dfrac{2\gamma}{r}$

B. $P_i - P_o = \dfrac{2\gamma}{r}$

C. $P_i - P_o = 2\gamma r$

#QDLE#Q#AB*C##



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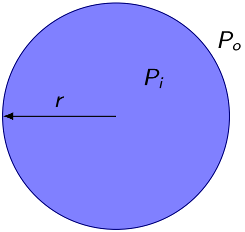


> Surface tension leads to pressure discontinuity at drop's interface

> Capillary pressure $\propto 1/r$ must be exceeded to inject fluid in a pore

Self-assessment – Laminar flows

What is/are the conditions to obtain a Stokes flow?

A. Negligible viscosity and stationarity

B. Negligible inertia and stationarity

C. High Reynolds number

D. Stationarity

#QDLE#Q#AB*CD##

What is/are the conditions to obtain a Stokes flow?

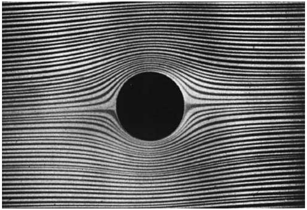
A. Negligible viscosity and stationarity

B. **Negligible inertia and stationarity**

C. High Reynolds number

D. Stationarity

- Reynolds number: inertia/viscosity competition $\left(Re = \dfrac{\rho U L}{\eta}\right)$
- Laminar flow ⇔ low Reynolds number
- Stokes equation : $\eta \Delta \overrightarrow{u} = \overrightarrow{\mathrm{grad}} P - \rho \overrightarrow{f}$



Self-assessment – Laminar flows

- ΔP: pressure difference
- L: tube length
- R: tube radius
- η: dynamic viscosity

What is the mean velocity of a laminar flow in a tube?

A. $\dfrac{\eta}{2}\dfrac{\Delta P}{L}R^2$

B. $\dfrac{1}{2\,\eta}\dfrac{\Delta P}{L}R^3$

C. $\dfrac{1}{8\,\eta}\dfrac{\Delta P}{L}R^2$

#QDLE#Q#ABC*##

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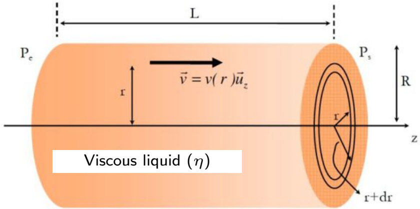

Credits: O. Granier

> $\vec{v} = \frac{\Delta P}{L}\frac{R^2}{4\,\eta}\left(1 - \frac{r^2}{R^2}\right)\vec{u_z}$ with $\Delta P = P_e - P_s$

Self-assessment – Mass balance

How do you perform a local mass balance?

A. Compute time variations of mass flux

B. Compute then balance mass flux and mass variations

C. I don't know

#QDLE#Q#AB*C##

How do you perform a local mass balance?

A. Compute time variations of mass flux
B. **Compute then balance mass flux and mass variations**
C. I don't know

> $\frac{d}{dt}$(Mass in elementary volume)= Mass flux **into** volume

- ρ: density
- t: time
- $\overrightarrow{u}$: velocity

And finally what do you get?

#QDLE#Q#A*BC##

A. $\dfrac{\partial \rho}{\partial t} + \operatorname{div}\left(\rho \overrightarrow{u}\right) = 0$

B. $\dfrac{\partial \rho}{\partial t} + \Delta\left(\rho \overrightarrow{u}\right) = 0$

C. $\dfrac{\partial \rho}{\partial t} = 0$

Self-assessment – Mass balance

- ρ: density
- t: time
- $\vec{u}$: velocity

And finally what do you get?

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C. $\dfrac{\partial \rho}{\partial t} = 0$

> Relation between mass and density

> Vectorial analysis

- $\vec{r}$: particles position
- t: time
- D: diffusion coefficient

What is the mean-square displacement of a diffusing particle (3D)?

#QDLE(#Q#ABC*##

A. $\langle \vec{r}(t) \rangle = 6Dt$

B. $\langle \vec{r}^{\,2}(t) \rangle = \sqrt{6Dt}$

C. $\langle \vec{r}^{\,2}(t) \rangle = 6Dt$

Self-assessment – Diffusion

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> Random diffusion due to thermal molecular move

> $D = \dfrac{k_b T}{3\pi d \eta}$ (Stokes-Einstein)

Self-assessment – Diffusion

- $\overrightarrow{j_{th}}$: thermal flux density
- λ: thermal conductivity
- T: temperature

What is the Fourier's law expression?

#QDLE#Q#AB*C##

A. $\overrightarrow{j_{th}} = \lambda \overrightarrow{\mathrm{grad}}\, T$

B. $\overrightarrow{j_{th}} = -\lambda \overrightarrow{\mathrm{grad}}\, T$

C. I don't know

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C. I don't know

> Thermal flux from high to low temperatures

> Combined with energy conservation (balance) to obtain heat equation

- Lecture decomposed in **five** parts
- Two central parts : Upscaling and Hydrodynamic transport

Main objectives At the end of these lectures, using the specific objectives of each part, you should be able to:

- describe a porous media using appropriate vocabulary;
- describe each kind of transfer qualitatively;
- develop the main steps of demonstrations for formulae demonstrated in the lecture;
- solve a complex problem including classic hydrodynamics, hydrostatics and transfers in porous media.

Lecture proceedings

- Description of porous media
  - $\sim$ 1.5 session
- Upscaling to porous media
  - $\sim$ 1.5 session
- Hydrodynamic transport in a porous medium
  - $\sim$ 3 sessions
- Diffusion and dispersion in a porous medium
  - $\sim$ 1.5 sessions
- Thermal transfer in a porous medium
  - $\sim$ 1.5 sessions
- Examination
  - January 26th
  - Made of two exercises: classical one and article analysis
  - All documents allowed (no electronic device)